My time at UConn - a 43 year Odyssey

Philip D. Mannheim

University of Connecticut

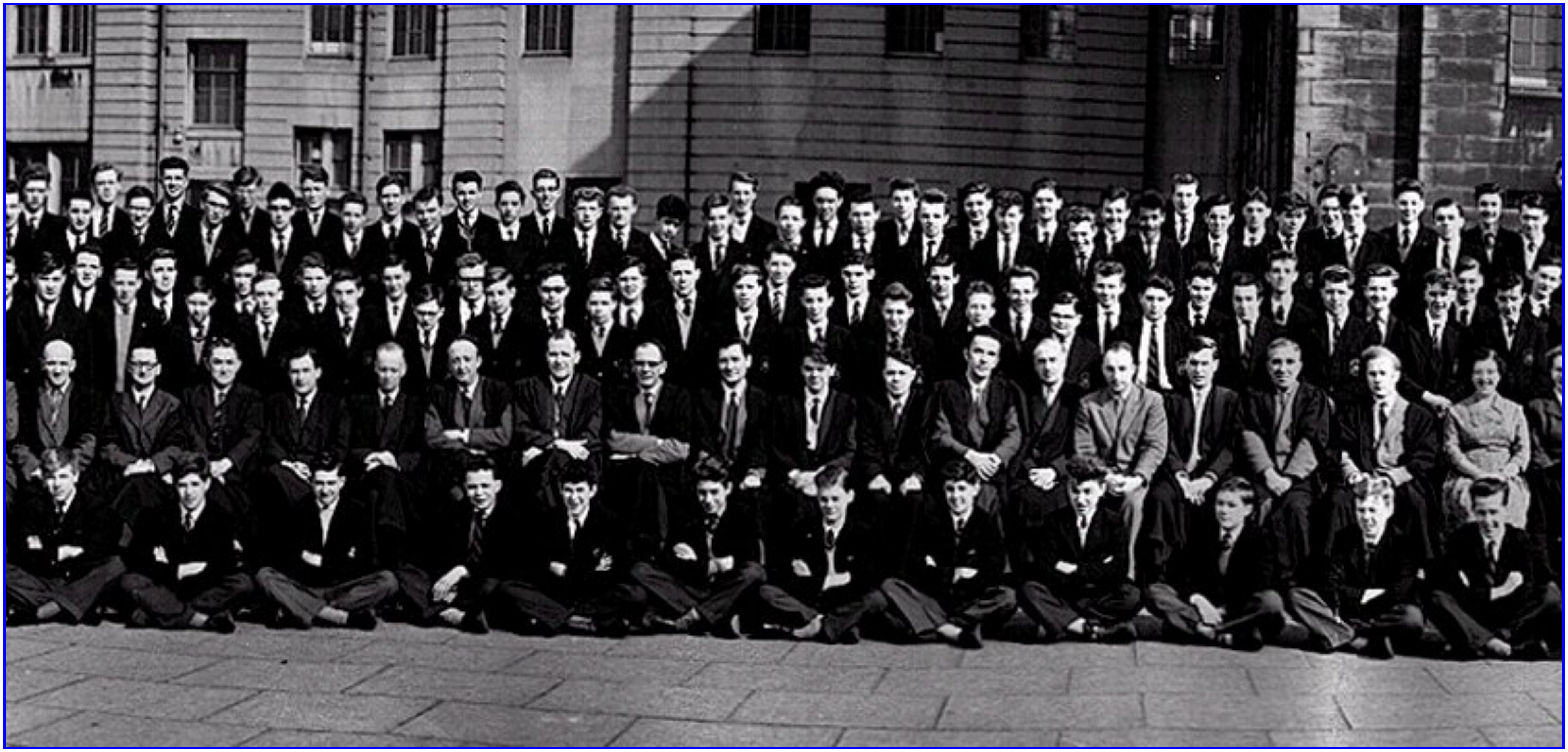
Colloquium at the University of Connecticut

April 2022

I arrived at UConn on September 1, 1979, and I will retire on July 1, 2022. In this talk I will describe some of the things that I did in between. I will describe some of my departmental and university activities, and will discuss some of the research I have done, research that benefited immeasurably from the supportive environment that my departmental colleagues and students have constantly provided me with.

Upper School, April 1960

Section 1 | Section 2 | Section 3 | Section 4 | Section 5



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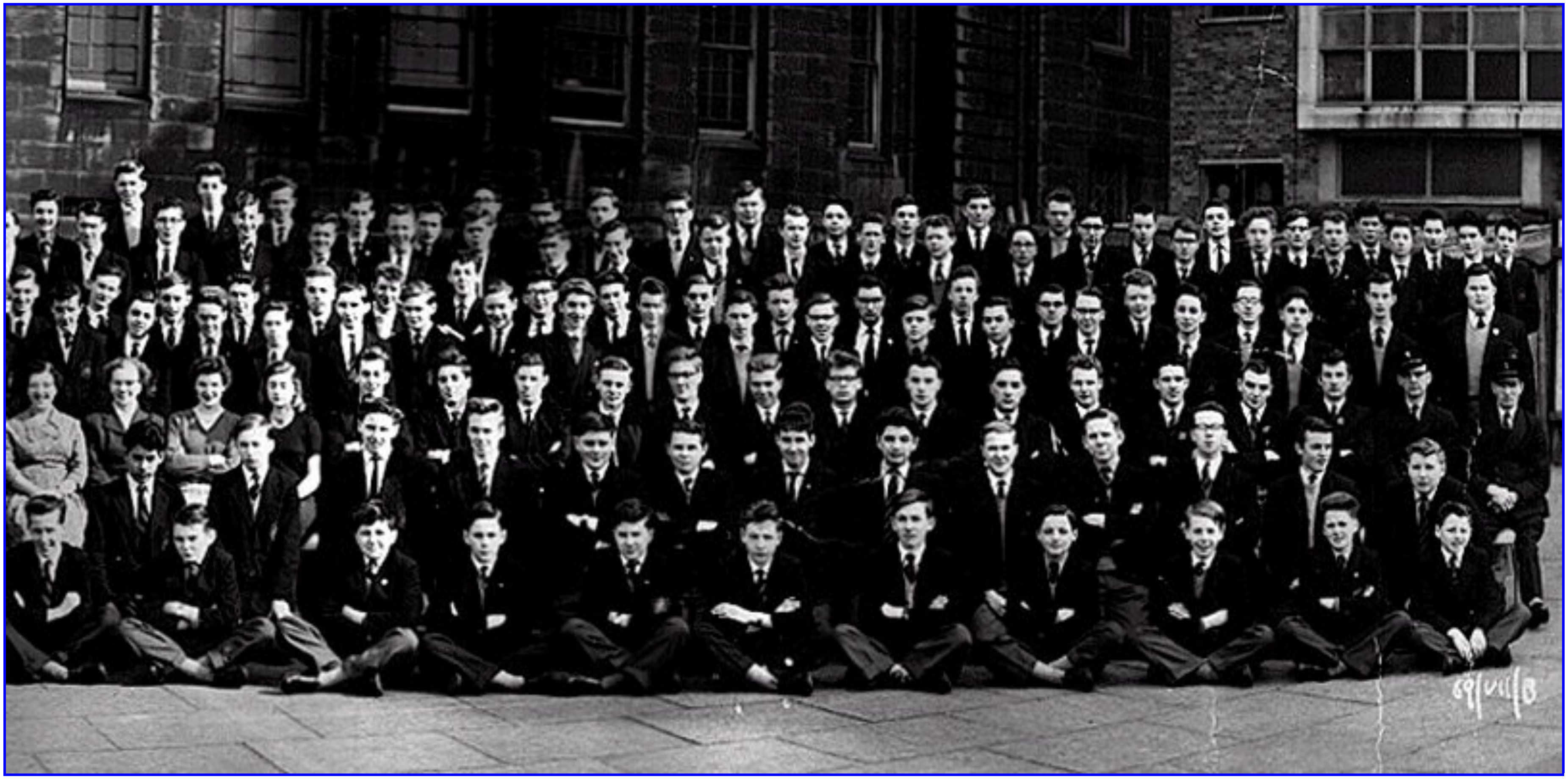
Upper School, April 1960

Section 1 | Section 2 | Section 3 | Section 4 | Section 5



Liverpool Institute High School for Boys

Mount Street, Liverpool. (1825-1985)

SEARCH

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he button below to
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Form M6A2, 1960/61

11 Scholarships to Oxbridge in one year!

9 of them in Mathematics.

Physics Dept. Office Copy

DEPARTMENT OF PHYSICS

ANNUAL REPORT

1978-79

II. INSTRUCTION

A. Enrollments

	Spr. '79	Fall '78	Spr. '78	Fall '77	Spr. '77	Fall '76
100						
100	922	1002	1013	1197	1123	1140
200	82	70	93	81	120	131
300	50	68	74	93	95	113
Total	1054	1140	1180	1371	1338	1384
Year Total	2194		2551		2722	

Average lecture section size

Spr. '79	Fall '78	Spr. '78	Fall '77	Spr. '77	Fall '76
29.5	26.3	36.0	40.5	40.5	44.3

Course	1978 Fall	1979 Spring
101	82	97
103	34	31
121	266	230
122	169	201
141-142	51	36
143	21	-
151	223	34
152	25	200
155	112	93
191	19	-
209-210	3	5
230	2	19
242-246	17	14
255-257	9	4
256	-	5
258-259	2	1
261-262	10	9
271	-	10
273	6	-
281	12	-
291	4	10
298	-	2
299	5	3

Course	1978 Fall	1979 Spring
300	2	2
308-324	6	6
311-312	14	6
314	3	3
318-319	10	8
321-306	1	9
322-323	8	7
328-329	1	-
335	-	6
337	11	-
338	-	3
341	3	-
342	4	-
343	5	-

II. D. Physics Majors

	1979	1978	1977	1976	1975
Seniors	5	10	14	19	11
Juniors	13	11	24	12	27
Total	18	21	38	31	38

E. Graduate Students

	1979	1978	1977	1976	1975
M.S.	26	19	23	25	18
Ph.D.	38	39	30	30	37
Total	64	58	53	55	55

F. Physics Degrees Awarded

	1979	1978	1977	1976	1975
Ph.D.	3	5	5	5	10
M.S.	5	9	12	11	8
B.S. & B.A.	5	10	12	17	11

See II.F. for lists of degree recipients and for placement of Master's and Ph.D. recipients.

III. RESEARCH AND PROFESSIONAL ACTIVITIES

A. Publications 1978-79 - Not previously listed.

Azaroff

Sabbatical Leave, 1978-79.

Bartram

Non-radiative de-excitation of deep centres, A. M. Stoneham and R. H. Bartram, Solid-State Electronics 21:1325-29 (1978).

Low-lying excited states of N_3^-, A. R. Rossi and R. H. Bartram, J. Chem. Phys. 70:532-37 (1979).

Fluorescence of potassium azide, P. J. Kemmey, R. H. Bartram, A. R. Rossi and P. W. Levy, J. Chem. Phys. 70:538-40 (1979).

Ab initio calculations of the electronic properties of N_4^- in KN_3, G. D. Bent and R. H. Bartram, J. Phys. Chem. Solids (accepted for publication) (1979).

Bedding

Measurement of first vibrational lifetime of LiCl, D. R. Bedding and T. I. Moran, Physical Review A. Scheduled for May or June edition.

Best

Bremsstrahlung isochromats and density of states: chromium, P. E. Best and C. C. Chu, Japanese Journ. of App. Phys. 17, Supp. 17-2, 317 (1978).

Bremsstrahlung isochromat of nickel: effects of beam energy and final state symmetry, C. C. Chu and P. E. Best, Phys. Rev. B1, May 1979.

Electronic structure of Si(111) - Cℓ by angle - resolved secondary emission and electron energy loss, P. E. Best, Phys. Rev. B19, 1054 (1979).

Energy and angular-dependent secondary electron emission from a silicon (111) 7 x 7 surface. II. Emission from surface resonances, P. E. Best, Phys. Rev., February 1979.

Best cont'd

Surface resonances on Cu(100), A. Price, P. Jennings, P. Best and J. Cornish. Submitted to Surface Science.

Budnick

Evidence for the angular dependence of the hyperfine interactions in complex magnetic spin structures, V. Niculescu and J. I. Budnick, Sol. St. Comm. 26, 607 (1978).

Electron transport in $Fe_{3+x}Si_{1-x}$, J. I. Budnick, W. B. Muir, V. Niculescu and K. Raj, Inst. Phys. Conf. Ser. No. 39, 196 (1978).

Magnetic ordering in exchange-enhanced $(Pd_{1-y}Ag_y)_{100-x}Fe_x$ and $(Pd_{1-y}Rh_y)_{100-x}Fe_x$ alloys, W. A. Hines, J. I. Budnick, A. H. Menotti, R. N. Paolino and T. J. Burch, Phys. Rev. B19, 338 (1979).

Relating structural, magnetic-moment, and hyperfine-field behavior to a local-environment model in $Fe_{3-x}Co_xSi$, V. Niculescu, J. I. Budnick, W. A. Hines, K. Raj, S. Pickart and S. Skalski, Phys. Rev. B19, 452 (1979).

Hyperfine field distribution in $Fe_3Si_{1-x}Al_x$ Alloys and a theoretical interpretation, T. J. Burch, J. I. Budnick, K. Raj, P. Jena and V. Niculescu, Phys. Rev., in press.

Damon

Post-use review of Physics by Paul Tipler, D. H. Damon, Am. J. Phys. 47, 478 (1979).

Foster

Applications of the Glauber and eikonal approximations to atomic collisions, to appear in Advances in Electronics and Electron Physics. (Paper co-authored by F. T. Chan, M. Lieber and W. Williamson, Jr.)

Gilliam

Trapped-hole centers associated with trivalent cations in tetragonal GeO_2, R. B. Bossoli, T. J. Welsh, O. R. Gilliam and M. Stapelbroek, Phys. Rev. B19, (1979). In press.

Nonaxial holelike defect in hydroxyapatite x-ray-irradiated at 6K, D. M. Close, M. M. Mengeot and O. R. Gilliam. To be submitted prior to July 1st of this year.

Hahn

Photoionization of positive ions, Y. Hahn, Phys. Lett. A67, 345 (1978).

Perturbed stationary method to ion-atom collisions, Y. Hahn, J. Phys. B11, 3221 (1978).

Auger and radiative transitions of high Rydberg states, Y. Hahn, Phys. Lett. A68, 197 (1978).

Distorted wave theory of e-ion collisions. II, Phys. Rev. A18, 1028 (1978).

Effect of N^* resonances and nucleon-nucleus scattering, Y. Hahn, Phys. Rev. C18, 2447 (1978).

Papers accepted:

Modified multiple scattering expansion with correlations, with J. Retter, Phys. Rev. C19, 1174 (1979).

Dielect. rec. of positive ions I, submitted, JQSRT, with J. Gau.

Dielect. rec. of positive ions II, submitted, JQSRT, with J. Gau and J. Retter.

Dielect. rec. of positive ions III, submitted, JQSRT, with J. Gau and J. Retter.

Scaling properties of diel. rec. rates for Ne seq, with J. Gau, R. Luddy and J. Retter.

Papers in preparation: (to be submitted for publ. by July 1979)

Scale-breaking behavior of the capture rate for Be-seq.

Relativistic corrections to the dielect. rec. rates.

Hahn cont'd

Pion-helium elastic scattering at medium energies.

Auger contribution to the e-ion impact ionization.

Pion absorption by He nucleus.

Haller

Lorentz transformations of observable and ghost particle states in quantum electrodynamics and in a massive gauge theory, K. Haller and R. B. Sohn, Journal of Math. Physics 19, 1589 (1978).

The Gupta-Bleuler condition and infrared coherent states, K. Haller, Phys. Rev. D18, 3045 (1978).

Forbidden ghost states in non-abelian gauge theories, R. G. Brickner and K. Haller, Physics Letters 78B, 601 (1978).

Gauge equivalence of the electrodynamics of charged bosons, R. Sohn and K. Haller, Journal of Math. Physics 18, 641 (1977).

Hayden

Radiative lifetimes and oscillator strengths for allowed intra L-shell transitions in multiply charged chlorine ions, J. P. Forester, D. J. Pegg, P. M. Griffin, G. D. Alton, S. B. Elston, H. C. Hayden, R. S. Thoe, C. R. Vane and J. J. Wright, Phys. Rev. A18, 1476-1480 (1978).

Hines

Relating structural, magnetic-moment, and hyperfine-field behavior to a local-environment model in $Fe_{3-x}Co_xSi$, V. Niculescu, J. I. Budnick, W. A. Hines, K. Ray, S. Pickart and S. Skalski, Phys. Rev. B19, 452 (1979).

Magnetic ordering in exchange-enhanced $(Pd_{1-y}Ag_y)_{100-x}F_x$ and $(Pd_{1-y}Rh_y)_{100-x}Fe_x$ alloys, W. A. Hines, J. I. Budnick, A. H. Menotti, R. H. Paolino and T. J. Burch, Phys. Rev. B19, 338 (1979).

Electronic structure of ferromagnetic Ni-Al solid solutions, D. M. Pease, L. V. Azaroff, C. K. Vaccaro and W. A. Hines, Phys. Rev. B19, 1576 (1979).

Holmberg

EPR and optical studies of γ-irradiated MgO:Ga, G. E. Holmberg, K. H. Lee and J. H. Crawford, Jr., Phys. Rev. B19, 2436 (1979).

Double-quantum EPR transition in AgCl:Ni, G. E. Holmberg, J. C. Hempel and L. M. Slifkin, J. Chem. Phys. 71, 15 September (1979).

Islam

Is a nucleon core being seen in high-energy elastic pp scattering at large momentum transfer?, M. M. Islam and G. W. Heines, Lett. al Nuovo Cimento 22, 441 (1978).

Evidence for a nucleon core from high energy pp elastic scattering, Phys. Rev. Letters (submitted for publication).

Kappers

Point defects in particle-irradiated single crystals of tetragonal GeO_2, L. A. Kappers, O. R. Gilliam and M. Stapelbroek, Phys. Rev. B17, 4199 (1978).

Determination of the speed of light by measurement of the beat frequency of internal laser modes, R. G. Brickner, L. A. Kappers and L. P. Lipschultz, Amer. J. Phys. (accepted for publication).

Crystal field splitting in rutile-structure oxides from correlation of ESR and optical results: V^{4+} in tetragonal GeO_2, D. P. Madacsi, L. A. Kappers and J. F. Houlihan, Phys. Stat. Sol. (b) (accepted for publication).

Radiation effects in berlinite, L. E. Halliburton, L. A. Kappers and A. F. Armington, Proc. of the 33rd Annual Frequency Control Symposium, (submitted for publication).

Kessel

Inelastic-energy-loss measurements of multiple N- and M-shell excitations in 0.3- to 1.2-MeV Xe^+-Xe collisions, (with Spicuzza and Antar) Phys. Rev. A18, 776-780 (1978).

Electron-loss cross sections, for 20-MeV Cl^{4+} and I^{5+} ions incident on thin gaseous targets: Experimental measurements and potential-model data analyses, (with Antar and ORNL associates) Phys. Rev. A18, 2459-2463 (1978).

Kessel cont'd

Absolute charge state yields of 20 MeV ^{127}I ions emerging from a gas stripper, (with Antar and ORNL associates) Nuclear Inst. and Meth. 150, 529-535 (1978).

Klemens

Deviations from Matthiessen's rule and the electronic thermal conductivity of alloys, P. G. Klemens, in "Thermal Conductivity 15", ed. by V. V. Mirkovich, p. 203, Plenum Press, New York, 1978.

Electrical resistivity of gold, R. A. Matula and P. G. Klemens, High Temperatures-High Pressures 10, 105 (1978).

Lipschultz

Determination of the speed of light by measurement of the beat frequency of internal laser modes, R. G. Brickner, L. A. Kappers and F. P. Lipschultz, submitted to American Journal of Physics.

Madacsi

Effects of dopants on efficiencies of TiO_2 photoanodes in photoelectrolytic cells, D. P. Madacsi and J. F. Houlihan, Materials Research Bulletin (to be published)

Crystal field splitting in rutile-structure oxides from correlation of ESR and optical results: V^{4+} in tetragonal GeO_2, D. P. Madacsi, L. A. Kappers and J. F. Houlihan, Physica Status Solidi (b) 91, K105-109 (1979).

Doped polycrystalline TiO_2 electrodes for the photo-assisted electrolysis of water, J. F. Houlihan, D. P. Madacsi, et.al., Materials Research Bulletin 13, 1205-1212 (1978).

Mallett

Higgs mechanism and the inverse Einstein-Infeld-Hoffman problem, R. L. Mallett, J. Math. Phys. (to be published).

Markowitz

Sabbatical Leave, 1978-79.

Moran

Measurement of anisotropy of interaction of LiF with inert gases, (with Bedding, and Menotti), Phys. Rev. A18, 1038.

Lifetime of first vibrational state of electronic ground state of LiCl, (with Bedding), scheduled for Phys. Rev. A, June 1979.

Pease

Electronic structure of ferromagnetic Ni-Al solid solutions, D. M. Pease, L. V. Azaroff, C. K. Vaccaro and W. A. Hines, Phys. Rev. B19, 1576 (1979).

Peterson

The physics of electrochromism--an advanced laboratory experiment (with J. Parlett and R. S. Crandall), to appear in Am. Journal of Physics. In press.

Photoemission and optical studies of TPP and MgTPP (with B. Wei); Photoemission studies of cytosine and thymine (with M. Tulpule, (in preparation).

Site specificity in alkaline phosphatase from kinetic and NMR experiments (with J. Coleman and J. Cheblowski), (in progress).

Pollack

Experimental test of a scaling law for ion-molecule collisions: Ne^{+}-D_2, 1.5-3.5 keV, N. Andersen, M. Vedder, A. Russek, and E. Pollack, J. Phys. B11, L493 (1978).

Energy loss scaling in 0.5-3.5 keV Ne^{+} and Ne collisions with H_2 and D_2, N. Andersen, M. Vedder, A. Russek, and E. Pollack, submitted to Phys. Rev.

Rawitscher

Effect of breakup on the spin dependence of the deuteron-nucleus interaction, G. H. Rawitscher and S. N. Mukherjee, Phys. Rev. Letts 40, 1486 (1978).

What one can learn from (e,e') experiments on polarized targets, G. H. Rawitscher, in Proceedings of the 1977 Bates Linac Summer Study, p. 393-413, ed. by A. Bernstein (unpublished).

Russek

Experimental test of a scaling law for ion-molecule collisions: Ne^+-D_2, 1.5-3.5 keV, N. Anderson, M. Vedder, A. Russek and E. Pollack, J. Phys. B11, L493-496 (1978).

Electron capture to D_3^- and D_3^0 repulsive states by D_3^+ in C_S, C. Cisneros, I. Alvarez, R. Garcia G., C. F. Barnett, J. A. Ray and A. Russek, Phys. Rev. A19, 631-640 (1979).

Excitation and charge transfer in low-energy N_a^+-Ne collisions, J. Østgaard Olsen, M. Barat, ch. Goussorgues, V. Sidis, N. Anderson, A. Russek, T. Andersen, J. Pommier and J. Augusti, accepted for publication in the April issue of The Physical Review.

Diffraction and angular momentum effects in semiclassical atomic scattering theory, A. Russek, accepted for publication in the June issue of The Physical Review.

Dissociating states of the H_3^- system, R. Garcia G., A. R. Rossi and A. Russek, accepted for publication in the June 15 issue of The Journal of Chemical Physics. (This work was Garcia's doctoral work under my supervision).

Energy loss scaling for electronically elastic ion-molecule collisions: $Ne^+ + D_2$, $Ne^+ + H_2$, and $Ne + D_2$, 0.5-3.5 keV, N. Anderson, M. Vedder, A. Russek and E. Pollack. Submitted to The Physical Review and in the refereeing process.

Investigation of nonadiabatic effects in molecular-hydrogen Rydberg states by electric field ionization, T. J. Morgan, C. F. Barnett, J. A. Ray and A. Russek, accepted for publication in July issue of The Physical Review.

Formation of positive ions in dissociative collisions of HD_2^+ in H_2, I. Alvarez, C. Cisneros, J. A. Ray, C. F. Barnett and A. Russek. Submitted to The Physical Review and in the refereeing process.

Russek cont'd

Article on "Atomic and Molecular Collisions" in The Encyclopedia of Physics. In press, Addison-Wesley.

Schor

Theoretical studies of the diffusion coefficient of charged bovine serum albumin molecule, R. Schor and E. Serrallach, J. Chem. Phys., p. 3012-3015 (1979).

Smith

Interference structure in ion-atom collisions, W. W. Smith and D. A. Clark, IEEE Transactions on Nuclear Science NS-26, p. 1042-1046 (1979).

Sohn

Lorentz transformations of observable and ghost particle states in quantum electrodynamics and in a massive gauge theory, with K. Haller, J. of Math. Phys. 19, 1589 (1978).

The interaction of Schrodinger electrons and photons, with K. Haller, submitted to Physical Review A.

Lecture notes in quantum field theory, with Y. Takahashi, in preparation.

III. B.

RESEARCH GRANTS

Research Grants Continued or Awarded During the period July 1, 1978 to June 30, 1979.

U.S. Army Grant #DAAG2976G0130
Term: January 19, 1976 - April 25, 1979 Project #213
Amount: $140,382.
Title: Low Energy Inelastic Atomic & Molecular Collisions
Directors: E. Pollack and W.W. Smith

National Science Foundation Grant #PHY76-10352
Term: June 1, 1976 - November 30, 1978 Project #215
Amt: $66,200.
Director: Q. Kessel

National Science Foundation Grant #INT 76-05772
Term: April 1, 1977 - June 30, 1979 Project #236
Amt: $4,500.
Title: Electron Capture by Molecular Ions in Single Collisions...
Director: A. Russek

Energy Research & Development Agency Grant #EG-77-S-02-4444
Term: August 1, 1977 - July 31, 1978 Project #247
Amt: $20,000.
Title: Deuteron-Nucleus Spin Orbit Tensor Optical Potential...
Director: G. Rawitscher

United Technologies Research Corporation
Term: September 1, 1977 - December 31, 1979 Project #249
Amt: $8,850.
Title: Electronic Properties of Fuel Cell Catalysts and Their Supports
Director: J.I. Budnick

United Technologies Research Corporation
Term: January 1, 1978 - to completion Project #258
Amount: $30,000.
Title: Terminal Modification of the Accelerator Facilities
Director: J.I. Budnick

Department of Energy Grant #EY-76-S-02-2276
Term: January 1, 1978 - December 31, 1978 Project #259
Amt: $50,000.
Title: Radiative Processes Assoc. with Collisions of Hot Electrons...
Director: Y. Hahn

United Technologies Research Corporation
Term: June 5, 1978 - December 31, 1978 Project #266
Amt: $4,800.
Title: Material Treatment and Analysis
Director: J.I. Budnick

United Technologies Research Corporation
Term: July 1, 1978 - September 15, 1978 Project #271
Amt: $3,000.
Title: Ion Implantation Services

Research Grants Continued:

Department of Energy Contract #EG 77-S-02-4444 A001
Term: August 1, 1978 - July 31, 1979
Amount: $22,140.
Title: Deuteron-Nucleus Spin Orbit Tensor Optical Potential...
Director: G. Rawitscher
Project #273

Department of Energy Contract #DE-AC02-79-ER10336 A
Term: November 1, 1978 - October 31, 1980
Amount: $35,000.
Title: Investigation in Particle and Field Theory
Director: K. Haller and M. Islam
Project #280

Department of Energy Contract #EY-76-S-02-2276
Term: January 1, 1979 - September 30, 1979
Amount: $41,000.
Title: Radiative Processes Assoc. with Collisions of Hot Electrons..
Director: Y. Hahn
Project #281

United Technologies Research Corporation
Term: February 1, 1979 - to completion
Amount: $15,000.
Title: Ion Implantation Studies
Director: J.I. Budnick
Project #282

United Technologies Research Corporation
Term: May 1, 1979 - June 15, 1979
Amount: $3,000.
Title: Implant Fatigue Specimens
Director: J.I. Budnick
Project #289

University of Connecticut Research Foundation Grants Continued or Awarded

Term: May, 1975 -December 31, 1979
Amount: $7,500.
Title: Vacuum UV Properties of PGA & TPP Compounds
Director: C.W. Peterson
Project #194

Term: April, 1975 - June 30, 1979
Amount: $15,295.
Title: Low Energy Inelastic Atomic and Molecular Collisions
Directors: E. Pollack and W. Smith
Project #216

Term: May, 1976 - November 30, 1979
Amount: $14,195.
Title: Correlation Studies on Structure Property Relations...
Director: J.I. Budnick
Project #217

Term: January, 1977 - December 31, 1978
Amount: $2,530.
Title: Investigation of the Use of Sea Water..
Director: D. Madacsi
Project #226

Second Semester

DATE	SPEAKER	TITLE
February 9, 1979	Professor R. K. Chang Dept. of Engineering & Applied science Yale University New Haven, CT 06520	"Resonance Raman Scattering in Semiconductors"
February 16, 1979	Professor Robert Schor Dept. of Physics Univ. of Connecticut Storrs, CT	"Protein Interactions and Translational Diffusion"
February 23, 1979	Professor B. T. Feld Physics Department Massachusetts Institute of Technology Cambridge, Massachusetts	"Are Quarks Real?"
February 27, 1979 (Special)	Professor Michael Salour Massachusetts Institute of Technology Cambridge, Massachusetts	"Time and Frequency Resolve Spectroscopy of Atoms, Molecules and Solids"
February 28, 1979 (Special)	Professor Michael S.Lubell Dept. of Physics Yale University	"Photons, Atoms, Electrons, and Spin"
March 2, 1979	Dr. John P. Delvaille Center for Astrophysics Harvard College Observatory/Smithsonian Astrophysical Observatory 60 Garden Street Cambridge, Massachusetts	"Extragalactic X-ray Astronomy from SAS-3"
March 7, 1979 (Special)	Dr. D. S. Hamilton Dept. of Physics Univ. of Southern California University Park Los Angeles, CA 90007	"Energy Transfer in Impurity Doped Insulators"
March 8, 1979 (Special)	Dr. James C. Bergquist National Bureau of Standards Boulder, Colorado	"Saturated Absorption Optica Ramsey Fringes"

THE UNIVERSITY OF CONNECTICUT

PHYSICS DEPARTMENT

EINSTEIN CENTENNIAL CELEBRATION

Monday, April 9, 1979, 3:30 P.M., IMS 20

"The Day Einstein Became Famous"

Dr. J. P. McEvoy
The American School of London
2-8 Loudoun Road, London

(General Interest Lecture)

Wednesday, April 11, 1979, 2:00 P.M., P-201

"The Geometry of Colors"

Dr. Joseph W. Weinberg
Department of Physics
Syracuse University
Syracuse, New York

Wednesday, April 11, 1979, 7:30 P.M., IMS 20

"Relativity and Cataclysmic Evolution"

Dr. Joseph W. Weinberg
Syracuse University

(General Interest Lecture)

Thursday, April 12, 1979, 2:00 P.M., P-201

"Progress Toward A Unified Field Theory"

Dr. Ronald L. Mallett
Physics Department
University of Connecticut

Refreshments and an informal discussion will follow each lecture.

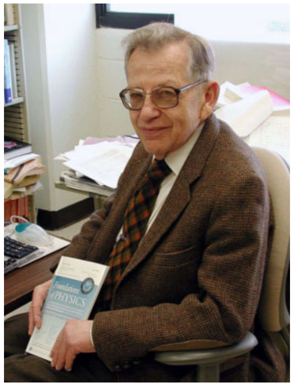
DEPARTMENT OF PHYSICS

1978 - 1979

THEORETICAL PHYSICS SEMINAR

DATE	SPEAKER	TITLE
June 14, 1978	Dr. Richard Sohn Physics Department University of Connecticut	"Spontaneous Symmetry Breaking and the Goldstone Boson"
June 23, 1978	Professor Y. Takahashi Physics Department University of Connecticut	"Chronological Products in Field Theory"
June 27, 1978	Professor K. Haller Physics Department University of Connecticut	"Infinite Range Problems in Identifying Goldstone Bosons
June 30, 1978	Dr. Ronald Mallett Department of Physics University of Connecticut	"Gravitational Field, Gauge Theories and Symmetry"
October 26, 1978 (Joint with Atomic)	Professor A. Russek Dept. of Physics Univ. of Connecticut	"Diffraction Effects in Semiclassical Scat-tering"
November 9, 1978 (Joint with Atomic)	Dr. Peter Milonni Perkin-Elmer Corp. Electro-Optical Div. 50 Danbury Road Wilton, CT 06897	"Is Quantum Mechanics Necessary in Optics?"
November 15, 1978 *November 29, 1978	Professor R. D. Koshel Physics Department University of Maryland	"A New Formulation of the Distorted Wave Approxi-mation"
December 6, 1978	Dr. Brian G. Kenny Theory Division Fermi Lab P.O. Box 500 Batavia, Illinois 60510	"Canonical Transformation in Classical and Quantum Mechanics"
	(Second Semester)	
February 6, 1979	Dr. Philip Mannheim Physics Department University of Oregon	"Phase Transitions in Particle Theory and Many-Body Theory"
*November 29, 1978	Professor K. S. Kang Physics Department Brown University	"Weak Neutral Current: Theory and Experiments"

The people responsible for hiring me



Kurt Haller

Munir Islam

Joe Budnick

University Activities

Academic make up day, Religious Holiday requirements, Sidewalks on North Eagleville, Devised university academic calendar (with 3 other faculty).

University Senate (10 years or so), University Senate Budget Committee (25 years or so). Member Advisory Committee for Judaic Studies and Contemporary Jewish Life, Faculty advisor to Chabad Jewish student group (43 years – a university record)

Probably proposed the largest number of failed motions at the University Senate. Failed to get covered sidewalks approved.

Departmental Activities

Introduced 5 courses: Introduction to Physics for first year physics majors, Mathematical Methods for physics majors (together with James O'Brien), Astrophysics and Modern Cosmology for physics majors and physics graduate students, General Relativity and Cosmology (part of Astronomy minor) for physics majors and physics graduate students, Modern Physics for physics graduate students.

Hosted 5 Katzenstein Endowment visits by Nobel Laureates: Gerard 't Hooft, David Gross, John Mather, Takaaki Kajita, Ray Weiss.

Chinese United States Physics Exchange (CUSPE) Program (helped Kurt Haller). The first two students were Deng Yaobing (my advisee) and Yang de-Ping (Bill Hines, Joe Budnick and Doug Pease).

Served as chair of PTR and Prelim Committees.

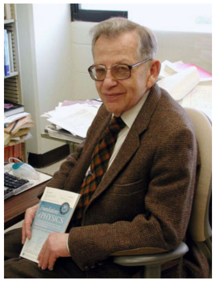
Organized 3 international conferences in Storrs: DPF 1988 (with Kurt Haller, Munir Islam, Dan Caldi, Ron Mallet and Mark Swanson), IARD 2006 (with Ron Mallet), IARD 2014 (with Ron Mallet and James O'Brien).

Edited an issue of Foundations of Physics honoring Kurt Haller's 65th birthday.

Established and maintained departmental journal library.

Set up the Fay and Philip Mannheim Endowment at the UConn Foundation for Physics Graduate Student Support.

Helped build up particle physics group. Principle Investigator on DOE particle physics grant. Group grant still in place after 44 years.



Kurt Haller

Munir Islam

Philip Mannheim

Mark Swanson

Daniel Caldi

Gerald Dunne

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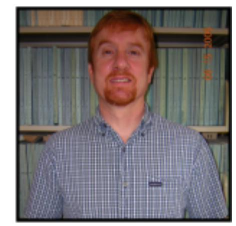

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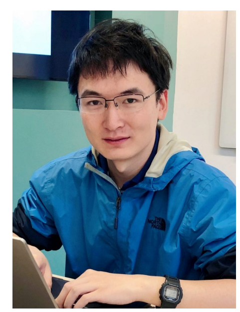

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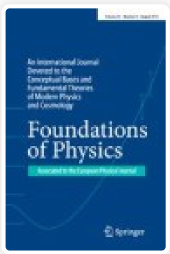

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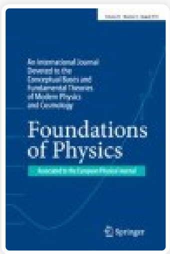

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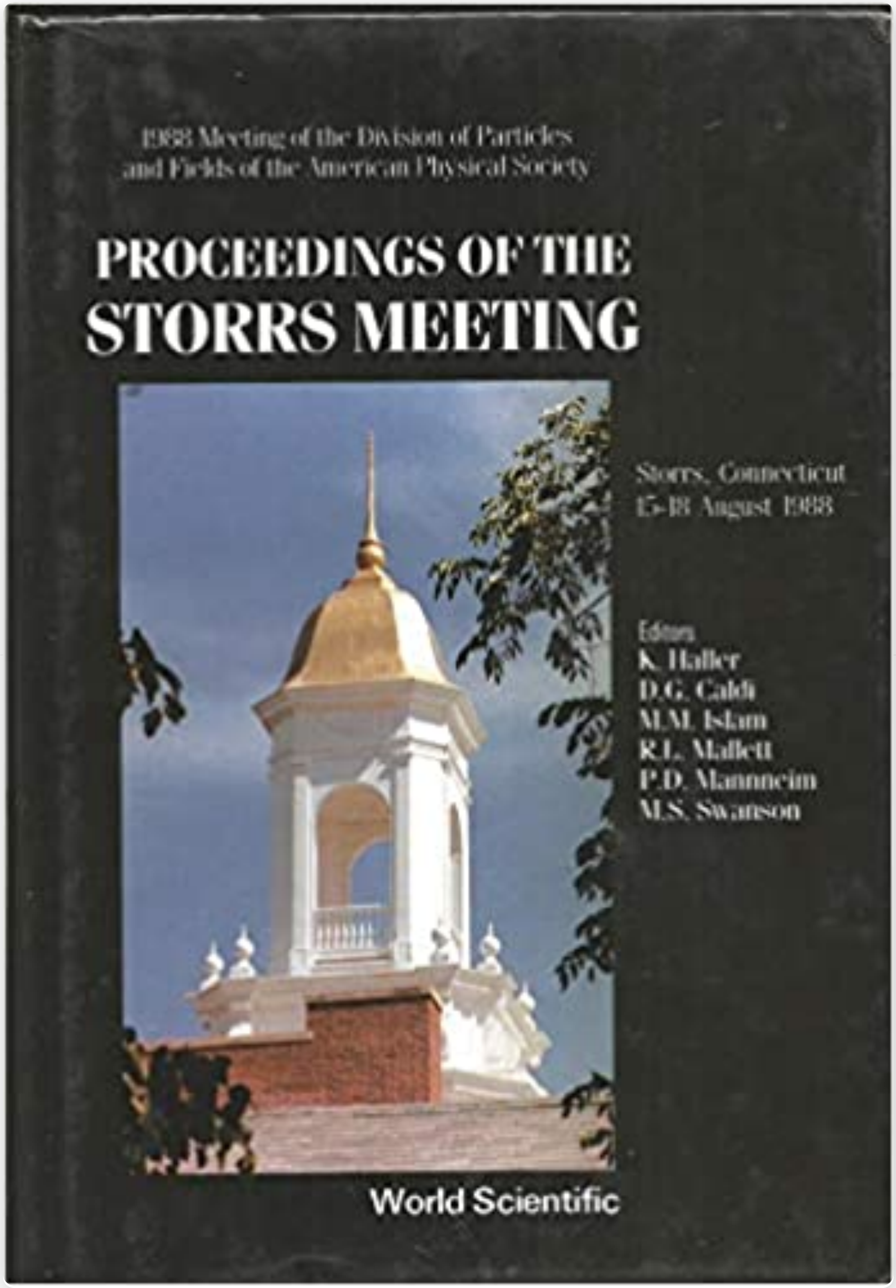
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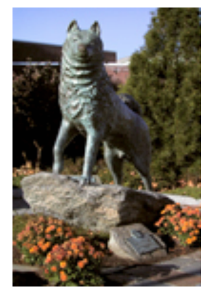

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The 9th Biennial Conference on Classical and Quantum Relativistic Dynamics of Particles and Fields

University of Connecticut (UConn) ♦ Storrs, CT, USA
9 - 13 June 2014

The International Association for Relativistic Dynamics (IARD) hosted its 9th biennial meeting at The University of Connecticut (UConn) Storrs, CT, USA, 9 - 13 June 2014.

IARD 2014 marks the association's 16th year of activity since its founding at the 1998 meeting and took place in conjunction with a conference on PT symmetry.

Eight previous meetings were held in Houston, Texas, Bar Ilan University in Tel Aviv, Israel, Howard University in Washington DC, Saas Fee, Switzerland, the University of Connecticut in Storrs, CT, Aristotle University in Thessaloniki, Greece, and Hualien, Taiwan, and Galileo Galilei Institute for Theoretical Physics (GGI) in Florence, Italy.

The goal of these meetings is to bring together researchers from diverse fields whose interests involve relativistic dynamics, both classical and quantum. The conference program aims to present recent developments in the abstract theoretical aspects of general approaches in quantum field theory, conformal field theories and string theories, manifestly covariant approaches to classical mechanics, quantum theory, and statistical mechanics, general relativity, classical and quantum gravity, and explore application in such areas as high energy electron spectroscopy, quark-gluon plasma generation in heavy ion collisions, general high energy scattering and particle decay, cosmology, gravitational waves, and relativistic quantum information.

IARD would like to express its gratitude to the University of Connecticut and to Nora Berrah, head of the Department of Physics, for hosting IARD/PT 2014.

We also express our appreciation to Philip Mannheim for his efforts as chair of the organizing committee, and to committee members James O'Brien and Ronald Mallet. At the IARD business meeting, James O'Brien was elected IARD Vice President. We are grateful to outgoing Vice President Ronald Mallet who has graciously offered to continue supporting the organization.

Conference Proceedings

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Einstein Centennial Celebration

The 2015/2016 academic year marked the centennial anniversary of Einstein's seminal development of the General Theory of Relativity in 1915. As part of the celebration that this occasioned throughout the worldwide physics community, the UConn Physics Department presented a series of special Einstein colloquia given by a very distinguished set of speakers, as organized by Professor Philip Mannheim and Professor Ronald Mallett. The kickoff talk, entitled "A 'Very' Brief History of General Relativity" was presented by Professor Mallett. In a joint presentation with the Women in Physics Series Professor Marla Geha (Director of Yale Telescope Resources) presented "The Darkest Galaxies". This talk was cohosted by Hope Whitelock a current UConn physics major. Professor Sam Werner (National Institute of Standards and Technology (NIST)) presented "The Effect of the Earth's Gravity and Rotation on the Quantum Mechanical Phase of the Neutron". Professor Robert Blum (Deputy Director National Optical Astronomy Observatory (NOAO)) presented "Astrophysics with Large Ground Based Surveys at the National Observatory". Professor Rainer Weiss (MIT and Laser Interferometry Gravitational Observatory (LIGO)) presented "A Brief History of Gravitational Waves: Theoretical Insight to Measurement". Professor Shep Doeleman (Haystack Observatory MIT) presented "The Event Horizon Telescope: Imaging and Time-Resolving a Black Hole". In a joint presentation with the UConn Center for Judaic Studies, Professor David Kaiser (Program in Science, Technology, and Society MIT) presented "Einstein's Legacy: Studying Gravity in War and Peace". The colloquium of Professor Weiss was particularly auspicious, because shortly after his visit the LIGO program that he jointly led announced the epochal first detection of the gravity waves that Einstein's General Theory of Relativity required. A most fitting centennial year discovery.

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128. P. D. Mannheim, P. Lowdon and S. J. Brodsky, *Comparing light-front quantization with instant-time quantization*, Physics Reports **891**, 1 (2021). (arXiv:2005.00109 [hep-ph])

129. P. D. Mannheim, *Exact solution to perturbative conformal cosmology in the recombination era*, Physical Review D **102**, 123535 (2020). (arXiv:2009.06841 [gr-qc])

130. A. Amarasinghe and P. D. Mannheim, *Cosmological fluctuations on the light cone*, Physical Review D **103**, 103517 (2021). (arXiv:2011.02440 [gr-qc])

131. A. Amarasinghe, T. Liu, D. A. Norman and P. D. Mannheim, *Exact solution to perturbative conformal cosmology from recombination until the current era*, Physical Review D **103**, 104022 (2021). (arXiv:2101.02608 [gr-qc])

132. P. D. Mannheim and J. W. Moffat, *External field effect in gravity*, International Journal of Modern Physics D **30**, 2142009 (2021). (arXiv: 2103.13972 [gr-qc])

133. P. D. Mannheim, *Antilinear symmetry and the ghost problem in quantum field theory*, Proceedings of Virtual Seminar Series on Pseudo-Hermitian Hamiltonians in Quantum Physics, IOP Conference Series **2038**, 012018 (2021). (arXiv: 2104.03708 [hep-th])

134. P. D. Mannheim, *Critique of the use of geodesics in astrophysics and cosmology*, arXiv:2105.08556 [gr-qc]

135. P. D. Mannheim, *Structure of conformal gravity in the presence of a scale breaking scalar field*, arXiv:2105.14679 [gr-qc]

136. P. D. Mannheim, *Extension of the Goldstone and the Englert-Brout-Higgs mechanisms to non-Hermitian theories*, arXiv:2109.08714 [hep-th]

137. P. D. Mannheim, *Solution to the ghost problem in higher-derivative gravity*, Proceedings of the Workshop on Quantum Gravity, Higher Derivatives and Nonlocality, Il Nuovo Cimento C, in press. (arXiv:2109.12743 [hep-th])

Philip David Mannheim – Conference Proceedings

1. A. Davidson, P. D. Mannheim and K. C. Wali, *Flavor-color-hypercolor unification based on* $SO(10)_V\ X\ SO(10)_H$, in Gauge theories, massive neutrinos, and proton decay, Proceedings of Orbis Scientiae, University of Miami, January 1981. Studies in the natural sciences Vol. **18**, edited by B. Kursunoglu and A. Perlmutter, Plenum Press, N. Y. (1981).

2. P. D. Mannheim, *Phonons: interactions with electromagnetic waves*, in Encyclopedia of Materials Science and Engineering, edited by M. Bever, Pergamon Press, N. Y. (1986).

3. P. D. Mannheim, *Symmetry and spontaneously broken symmetry in the physics of elementary particles*, Computers and Mathematics with Applications **12B**, 169 (1986). Republished in Symmetry: unifying human understanding, International series in modern applied mathematics and computer science Vol. 10, edited by I. Hargittai, Pergamon Press, N.Y. (1986).

4. P. D. Mannheim and D. Kazanas, *Exact vacuum solution to fourth order Weyl gravity*, in Proceedings of the Storrs meeting, the fourth meeting (new series) of the Division of Particles and Fields of the American Physical Society, University of Connecticut, August 1988. Edited by K. Haller, D. C. Caldi, M. M. Islam, R. L. Mallett, P. D. Mannheim, and M. S. Swanson, World Scientific Press, Singapore (1989).

5. K. Haller, D. C. Caldi, M. M. Islam, R. L. Mallett, P. D. Mannheim and M. S. Swanson, joint editors, *Proceedings of the Storrs meeting*, the fourth meeting (new series) of the Division of Particles and Fields of the American Physical Society, University of Connecticut, August 1988. Published by World Scientific Press, Singapore (1989).

6. P. D. Mannheim, *Some exact solutions to conformal Weyl gravity*, in Nonlinear Problems in Relativity and Cosmology, Proceedings of the Sixth Florida Workshop on Nonlinear Astronomy, University of Florida, October 1990. Edited by J. R. Buchler, S. L. Detweiler, and J. R. Ipser, Annals of the New York Academy of Sciences, Vol. **631**, 194 (1991).

7. P. D. Mannheim and D. Kazanas, *Current status of conformal Weyl gravity*, in Proceedings of the "After the First Three Minutes" Workshop, University of Maryland, October 1990. A. I. P. Conference Proceedings No. **222**, edited by S. S. Holt, C. L. Bennett, and V. Trimble, A. I. P., N. Y. (1991).

8. D. Kazanas and P. D. Mannheim, *Dark matter or new physics?*, in Proceedings of "After the First Three Minutes", University of Maryland, October 1990. A. I. P. Conference Proceedings No. **222**, edited by S. S. Holt, C. L. Bennett, and V. Trimble, A. I. P., N. Y. (1991).

9. P. D. Mannheim, *Conformal gravity, cosmology and Newton's law*, in Proceedings of the XXth International Conference on Differential Geometric Methods in Theoretical Physics, Baruch College/CUNY, New York, June 1991. Edited by S. Catto and A. Rocha, World Scientific Press, Singapore (1992).

10. P. D. Mannheim, *Four dimensional conformal gravity, confinement, and galactic rotation curves*, in Proceedings of "PASCOS 94", the Fourth International Symposium on Particles, Strings and Cosmology, Syracuse, New York, May 1994. Edited by K. C. Wali, World Scientific Press, Singapore (1995). (gr-qc/9407010)

11. P. D. Mannheim and D. Kazanas, *Higgs mechanism and the structure of the energy-momentum tensor in Einstein gravity and conformal gravity*, in Proceedings of the Seventh Marcel Grossmann Meeting on General Relativity, Stanford, California, July 1994. Edited by R. T. Jantzen, G. M. Keiser and R. Ruffini, World Scientific Press, Singapore (1996). (gr-qc/9409050)

12. P. D. Mannheim, *Microlensing, Newton-Einstein gravity, and conformal gravity*, in Proceedings of "Dark Matter", University of Maryland, October 1994. A. I. P. Conference Proceedings No. **336**, edited by S. S. Holt and C. L. Bennett, A. I. P., N. Y. (1995). (astro-ph/9412007)

13. P. D. Mannheim, *Linear potentials in the cores of clusters of galaxies*, in Proceedings of "Clusters, Lensing, and the Future of the Universe", University of Maryland, June 1995. Astronomical Society of the Pacific Conference Series, Vol. **88**, edited by V. Trimble and A. Reisenegger, A. S. P., San Francisco (1996). (astro-ph/9508045)

14. P. D. Mannheim, *Dark matter: a challenge to standard gravity or a warning?*, in Proceedings of "Eighteenth Texas Symposium on Relativistic Astrophysics", University of Chicago, December 1996. Edited by A. V. Olinto, J. A. Frieman, and D. N. Schramm, World Scientific Press, Singapore (1998). (astro-ph/9701128)

15. P. D. Mannheim, *Imprint of the global Hubble flow on galactic rotation curves*, in Proceedings of "Galactic Halos: A UC Santa Cruz Workshop", University of California at Santa Cruz, August 1997. Astronomical Society of the Pacific Conference Series, Vol. **136**, edited by D. Zaritsky, A. S. P., San Francisco (1998). (astro-ph/9712090)

16. P. D. Mannheim, *How we got into the dark matter fix and how we can get out*, in Proceedings of "PASCOS 98", the Sixth International Symposium on Particles, Strings and Cosmology, Northeastern University, Boston, March 1998. Edited by P. Nath, World Scientific Press, Singapore (1999). (astro-ph/9807122)

17. P. D. Mannheim, *The equivalence principle in classical mechanics and quantum mechanics*, in "Contemporary Fundamental Problems". Edited by Valeri Dvoeglazov, Nova Science Publishers, New York (1999). (gr-qc/9810087)

18. P. D. Mannheim, *Curvature, galactic dynamics and cosmic repulsion*, in Proceedings of the International Conference on Galaxy Dynamics, Rutgers University, August 1998. Astronomical Society of the Pacific Conference Series, Vol. **182**, edited by D. Merritt, J. A. Sellwood and M. Valluri, A. S. P., San Francisco (1999). (astro-ph/9811256)

19. G. V. Dunne, L. P. Horwitz, M. M. Islam, and P. D. Mannheim, joint guest editors of 3 special monthly festschrift sections of Foundations of Physics in honor of the Seventieth Birthday of Kurt Haller, Foundations of Physics, **30**, Nos 3, 4, 5 (2000).

20. P. D. Mannheim, *Conformal gravity and a naturally small cosmological constant*, Proceedings of "20th Texas Symposium on Relativistic Astrophysics", Austin, December 2000, J. C. Wheeler and H. Martel (Eds.), American Institute of Physics, NY (2001). (astro-ph/9901219)

21. P. D. Mannheim, *Cosmic acceleration and a natural solution to the cosmological constant problem*, Proceedings of "The Role of Neutrinos, Strings, Gravity and Variable Cosmological Constant in Elementary Particle Physics", Coral Gables Conference, December 2000, B. N.

Kursunoglu, S. L. Mintz and A. Perlmutter (Eds.), Kluwer Academic/Plenum Publishers, NY (2001). (gr-qc/9903005)

22. P. D. Mannheim, *Is cosmic acceleration really recent?*, Proceedings of "Cosmology and Elementary Particle Physics", Coral Gables Conference, December 2001, B. N. Kursunoglu, S. L. Mintz and A. Perlmutter (Eds.), American Institute of Physics, NY (2002). (astro-ph/0204202)

23. P. D. Mannheim, *Localization issues for Robertson-Walker branes*, Proceedings of "Cosmology and Elementary Particle Physics", Coral Gables Conference, December 2001, B. N. Kursunoglu, S. L. Mintz and A. Perlmutter (Eds.), American Institute of Physics, NY (2002). (arXiv:0807.3685 [hep-th])

24. P. D. Mannheim, *Options for cosmology at redshifts above one*, Proceedings of "Short Distance Behavior of Fundamental Interactions", Coral Gables Conference, December 2002, B. N. Kursunoglu, M. Camcigil, S. L. Mintz and A. Perlmutter (Eds.), American Institute of Physics, NY (2003). (astro-ph/0302362)

25. P. D. Mannheim, *The work of Behram Kursunoglu*, Proceedings of "The Launching of La Belle Epoque of High Energy Physics and Cosmology", Coral Gables Conference, December 2003, T. Curtright, S. Mintz and A. Perlmutter (Eds.), World Scientific Publishing Company, Singapore (2004). (gr-qc/0405035)

26. P. D. Mannheim, *Dark matter and dark energy – fact or fantasy*, Montreal-Rochester-Syracuse-Toronto Conference, Montreal, Canada, May 2004. International Journal of Modern Physics **A19**, 5333 (2004).

27. P. D. Mannheim, *Causality in the brane world*, Presentation at the 26th International Colloquium on Group Theoretical Methods in Physics, New York City, June 2006. (hep-th/0607041)

28. P. D. Mannheim, *Dynamical symmetry breaking and the cosmological constant problem*, Proceedings of the 34th International Conference in High Energy Physics (ICHEP08), Philadelphia, 2008, eConf C080730. (arXiv:0809.1200 [hep-th])

29. P. D. Mannheim, *Why do we believe in dark matter and dark energy – and do we have to?*, in "Questions of Modern Cosmology – Galileo's Legacy", M. D'Onofrio and C. Burigana (Eds.), Springer Publishing Company, Heidelberg (2009).

30. P. D. Mannheim, *Conformal Gravity Challenges String Theory*, Proceedings of the Second Crisis in Cosmology Conference, CCC-2, Astronomical Society of the Pacific Conference Series Vol. 413. (F. Potter, Ed.), San Francisco (2009). (arXiv:0707.2283 [hep-th])

31. P. D. Mannheim, *Intrinsically quantum-mechanical gravity and the cosmological constant problem*, to appear in Proceedings of the International Conference on Two Cosmological Models, Universidad Iberoamericana, Mexico City, November, 2010 (J. Auping-Birch and A. Sandoval-Villalbazo, Eds.). (arXiv:1005.5108 [hep-th])

32. P. D. Mannheim, *Making the case for conformal gravity*, Proceedings of the International Conference on Two Cosmological Models, Universidad Iberoamericana, Mexico City, November, 2010 (J. Auping-Birch and A. Sandoval-Villalbazo, Eds.). (arXiv: 1101.2186 [hep-th])

33. P. D. Mannheim, *CPT symmetry without Hermiticity*, Proceedings of ICHEP2016, the 38th International Conference on High Energy Physics, Chicago, August 2016. (arXiv:1611.02100 [hep-th])

Philip David Mannheim – Monographs

1. P. D. Mannheim, *Brane-localized gravity*, Full length monograph, World Scientific Publishing Company, Singapore (2005). (http://www.worldscibooks.com/physics/5975.html)

Brane-Localized Gravity

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Pages: 356

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Philip David Mannheim – Recent Seminars

2000

1. Cosmic acceleration as the solution to the cosmological constant problem, seminar presented at MIT at joint MIT-CFA-Tufts cosmology seminar, February, 2000.
2. Conformal gravity I, seminar presented at MIT, March, 2000.
3. Conformal gravity II, seminar presented at MIT, March, 2000.
4. Cosmic acceleration as the solution to the cosmological constant problem, seminar presented at Brandeis University, March, 2000.
5. How good is Newton's law of gravity?, colloquium presented at University of Miami, April, 2000.
6. How we got into the dark matter fix and how we can get out, seminar presented at MIT, May, 2000.
7. Cosmic acceleration as the solution to the cosmological constant problem, seminar presented at Boston University, May, 2000.
8. Conformal gravity III, seminar presented at MIT, May, 2000.
9. Brane-localized gravity: could there be a macroscopically sized fifth dimension?, seminar presented at University of Connecticut, Storrs, September, 2000.
10. Dynamical localization of gravity, seminar presented at University of Connecticut, Storrs, September, 2000.
11. The crystal impurity problem and the Mossbauer effect, seminar presented at Argonne National Laboratory, November, 2000.
12. Gravitationally induced quantum interference, seminar presented at Argonne National Laboratory, November, 2000.
13. Brane-localized gravity, seminar presented at Argonne National Laboratory, November, 2000.
14. Cosmic acceleration and a natural solution to the cosmological constant problem, conference seminar presented at Orbis Scientiae 2000, Fort Lauderdale, Florida, December 2000.
15. Conformal gravity and a naturally small cosmological constant, poster presentation at 20th Texas Symposium on Relativistic Astrophysics, Austin, Texas, December 2000.

2001

1. The crystal impurity problem and the Mossbauer effect, seminar presented at University of Connecticut, Storrs, January, 2001.

2. Brane cosmology, seminar presented at MIT, February, 2001.
3. Cosmic acceleration as the solution to the cosmological constant problem, seminar presented at Texas A and M, March, 2001.
4. How we got into the dark matter fix and how we can get out, colloquium presented at University of Texas, March, 2001.
5. How recent is cosmic acceleration?, seminar presented at University of Connecticut, Storrs, May, 2001.
6. Recent developments in the crystal impurity problem, seminar presented at Argonne National Laboratory, Argonne, July 2001.
7. How recent is cosmic acceleration?, seminar presented at Argonne National Laboratory, Argonne, July 2001.
8. Gravitationally induced quantum interference, colloquium presented at Argonne National Laboratory, November 2001.
9. Localization issues for Robertson-Walker branes, conference seminar presented at Coral Gables 2001, Fort Lauderdale, Florida, December 2001.
10. Is cosmic acceleration really recent?, conference seminar presented at Coral Gables 2001, Fort Lauderdale, Florida, December 2001.

2002

1. How recent is cosmic acceleration?, seminar presented at Yale University, April 2002.
2. How we got into the dark matter fix and how we can get out, seminar presented at University of St. Andrews, St. Andrews Scotland, August 2002.
3. How we got into the dark energy fix and how we can get out, seminar presented at University of St. Andrews, St. Andrews Scotland, August 2002.
4. How recent is cosmic acceleration?, invited presentation at Cross-Channel Conference, Plymouth, England, August 2002.
5. The accelerating universe, colloquium presented at Daytona Beach Community College, Daytona, November 2002.
6. How we got into the dark matter fix and how we can get out, seminar presented at SLAC, Stanford University, December 2002.
7. Living with a large cosmological constant, seminar presented at SLAC, Stanford University, December 2002.
8. How recent is cosmic acceleration?, seminar presented at LBL, Berkeley, December 2002.

9. The accelerating universe, colloquium presented at San Francisco State University, December 2002.

10. Options for cosmology at redshifts above one, invited presentation at Coral Gables 2002 Conference, Fort Lauderdale, Florida, December 2002.

2003

1. How recent is cosmic acceleration?, seminar presented at Liverpool University, England, January 2003.

2. How we got into the dark matter fix and how we can get out, seminar presented at University of Florida, Gainesville, April 2003.

3. Living with a large cosmological constant, seminar presented at University of Florida, Gainesville, April 2003.

4. Living with a large cosmological constant, invited presentation at the 8th Wigner symposium, CUNY, New York City, May 2003.

5. Dark matter and dark energy - fact or fantasy, colloquium presented at University of Oxford, Oxford, England, August 2003.

6. Living with a large cosmological constant, seminar presented at University of Oxford, Oxford, England, August 2003.

7. Living with a large cosmological constant, invited seminar presented at Cosmo-03 Conference, Ambleside, England, August 2003.

8. The work of Behram Kursunoglu, invited seminar presented at Coral Gables Conference, Fort Lauderdale, December 2003.

9. Dark matter and dark energy - fact or fantasy, invited seminar presented at Coral Gables Conference, Fort Lauderdale, December 2003.

2004

1. Dark matter and dark energy - fact or fantasy, seminar presented at University of Cardiff, Cardiff, Wales, January 2004.

2. Dark matter and dark energy - fact or fantasy, seminar presented at University of Bristol, Bristol, England, January 2004.

3. Dark matter and dark energy - fact or fantasy, colloquium presented at Louisiana State University, Baton Rouge, January 2004.

4. The most expensive way to measure the velocity of sound, seminar presented at University of Connecticut, Storrs, February 2004.

5. Dark matter and dark energy - fact or fantasy, invited seminar presented at MRST Conference, Montreal, Canada, May 2004.

6. Dark matter and dark energy – fact or fantasy, seminar presented at Ben Gurion University of the Negev, Beersheva, Israel, July 2004.
7. Bounds on localized modes in the crystal impurity problem, seminar presented at the Budnickfest, University of Connecticut, Storrs, September 2004.
8. Quantizing acceleration dependent Lagrangians, seminar presented at the University of Connecticut, Storrs, November 2004.

2005

1. Dark matter and dark energy – fact or fantasy, seminar presented at University of Lancaster, Lancaster, England, March 2005.
2. Dark matter and dark energy – fact or fantasy, seminar presented at Rockefeller University, New York, May 2005.
3. Bounds on localized modes in the crystal impurity problem, seminar presented at Argonne National Laboratory, October 2005.
4. Alternatives to dark matter and dark energy, seminar presented at Kavli Institute, University of Chicago, October 2005.
5. Dark matter and dark energy – fact or fiction, seminar presented at University of Connecticut, Storrs, November 2005.
6. Dark matter and dark energy – fact or fiction, colloquium presented at Perimeter Institute, Waterloo, Canada, November 2005.
7. Dark matter and dark energy – fact or fiction, seminar presented at University of Toronto, Canada, November 2005.
8. Bounds on localized modes in the crystal impurity problem, seminar presented at University of Connecticut, Storrs, November 2005.
9. Dark matter and dark energy – fact or fiction, seminar presented at University of Liverpool, England, December 2005.
10. Dark matter and dark energy – fact or fiction, seminar presented at University of Manchester, England, December 2005.

2006

1. Gauge invariant treatment of the energy of a gravitational wave, seminar presented at University of Connecticut, Storrs, January 2006.
2. Dark matter and dark energy – fact or fiction, seminar presented at Imperial College, London, England, February 2006.
3. Dark matter and dark energy – fact or fiction, seminar presented at Fermilab, March 2006.

4. Causality in the brane world, seminar presented at Fermilab, March 2006.
5. Conformal gravity and dark energy, invited plenary presentation at Alternative Gravities and Dark Matter Workshop, Edinburgh, Scotland, April 2006.
6. Dark matter and dark energy – fact or fiction, seminar presented at Syracuse University, May 2006.
7. Cosmology in the Dvali-Gabadaze-Porrati brane world, seminar presented at Cornell/Syracuse Dark Energy Workshop, Syracuse University, May 2006.
8. Dark matter and dark energy – fact or fiction, seminar presented at Cornell University, May 2006.
9. Solution to the ghost problem in fourth order derivative theories, seminar presented at the 2006 Biennial Meeting of the International Association for Relativistic Dynamics, Storrs, May 2006.
10. Grassmann extension of the Stuckelberg proper time formulation of quantum mechanics, seminar presented at the 2006 Biennial Meeting of the International Association for Relativistic Dynamics, Storrs, May 2006.
11. Causality in the brane world, seminar presented at the 26th International Colloquium on Group Theoretical Methods in Physics, New York City, June 2006.
12. Dark matter and dark energy – fact or fiction, seminar presented at University of Pennysylvania, November 2006.
13. The 2006 Nobel prize in physics, colloquium presented at University of Connecticut, Storrs, November 2006.
14. Dark matter and dark energy – fact or fiction, seminar presented at McGill University, Montreal, November 2006.
15. Einstein's incredible legacy, colloquium presented at Bishop's University, Sherbrooke, Quebec, November 2006.
16. Einstein's incredible legacy, colloquium presented at Stockton College, Pomona, New Jersey, December 2006.
17. Causality and completeness issues in the brane world, seminar presented at New York University, December 2006.
18. Intoduction to the brane world, seminar presented at SUNY Stony Brook, December 2006.
19. Alternatives to dark matter and dark energy, seminar presented at SUNY Stony Brook, December 2006.
20. Causality and completeness issues in the brane world, seminar presented at the Miami 2006 Conference, Fort Lauderdale, December 2006.

2007

1. Dark matter and dark energy – fact or fiction, invited talk presented at the ”Open Questions for the Standard Cosmological Model” conference, Imperial College London, March 2007.

2. Dark matter and dark energy – fact or fiction, seminar presented at Los Alamos National Laboratory, May 2007.

3. Conformal gravity challenges string theory, invited talk presented at Pascos-07 symposium, Imperial College London, July 2007.

4. Dark matter and dark energy – fact or fiction, colloquium presented at University of Kansas, October 2007.

5. Dark matter and dark energy – fact or fiction, seminar presented at University of Wisconsin, October 2007.

6. Conformal gravity challenges string theory, seminar presented at University of Minnesota, October 2007.

7. Conformal gravity challenges string theory, invited talk presented at the Miami 2007 Conference, Fort Lauderdale, December 2007.

2008

1. Dark matter and dark energy – fact or fiction, seminar presented at University of Durham, Durham UK, February 2008.

2. Conformal gravity challenges string theory, seminar presented at University of Oxford, Oxford, February 2008.

3. Conformal gravity challenges string theory, seminar presented at California Institute of Technology, Pasadena, March 2008.

4. Conformal gravity challenges string theory, seminar presented at University of California at Los Angeles, March 2008.

5. Einstein’s incredible legacy, colloquium presented at Fresno State University, Fresno, California, March 2008.

6. Quantum Mechanics off the beaten track, colloquium presented at Washington University, St. Louis, April 2008.

7. Dark matter and dark energy – fact or fiction, colloquium presented at University of Nebraska, May 2008.

8. Conformal gravity challenges string theory, invited talk presented at Pascos-08 symposium, Perimeter Institute, Waterloo Canada, June 2008.

9. Conformal gravity challenges string theory, invited talk presented at the 34th International Conference on High Energy Physics (ICHEP08), Philadelphia, July 2008.

10. Does the cosmological constant problem presage a paradigm shift in gravitational theory?, invited talk at the Second Crisis in Cosmology Conference, Port Angeles, WA, September 2008.

11. Doing physics with non-diagonalizable Hamiltonians and the solution to the ghost problem in fourth-order derivative theories, seminar presented at Syracuse University, September 2008.

12. Quantum mechanics off the beaten track, colloquium presented at University of Connecticut, Storrs, October 2008.

13. The 2008 Nobel prize in physics, seminar presented at University of Connecticut, Storrs, November 2008.

14. Doing physics with non-diagonalizable Hamiltonians and the solution to the ghost problem in fourth-order derivative theories, seminar presented at Perimeter Institute, Waterloo Canada, November 2008.

15. Doing physics with non-diagonalizable Hamiltonians and the solution to the ghost problem in fourth-order derivative theories, invited talk presented at Miami 2008 Conference, Fort Lauderdale, December 2008.

2009

1. Does the cosmological constant problem presage a paradigm shift in gravitational theory?, invited talk at Intertwining Theory and Observational Evidence in Contemporary Cosmology, Wuppertal Germany, February 2009.

2. Doing physics with non-diagonalizable Hamiltonians and the solution to the ghost problem in fourth-order derivative theories, invited talk presented at Quantum Mechanics in the Complex Domain, St. Louis, March 2009.

3. Doing physics with non-diagonalizable Hamiltonians and the solution to the ghost problem in fourth-order derivative theories, seminar at the University of Chicago, April 2009.

4. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, seminar at University of Pennsylvania, September 2009.

5. Quantum Gravity, colloquium at the University of Connecticut, October 2009.

6. Quantum Gravity – The Details, seminar at the University of Connecticut, October 2009.

7. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, invited talk presented at the Miami 2009 Conference, Fort Lauderdale, December 2009.

2010

1. Quantum Conformal Gravity and Grandunification, seminar at Syracuse University, February 2010.

2. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, seminar at Syracuse University, February 2010.

3. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, seminar at University of Minnesota, February 2010.

4. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, seminar at University of Wisconsin, February 2010.

5. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, seminar at Imperial College, London, March 2010.

6. Classical Aspects of the Dark Matter and Dark Energy Problems, seminar at SUNY Stony Brook, April 2010.

7. Quantum Aspects of the Dark Matter and Dark Energy Problems, seminar at SUNY Stony Brook, April 2010.

8. Comprehensive Solution to the Cosmological Constant, Zero-Point Energy, and Quantum Gravity Problems, seminar at Brookhaven National Laboratory, April 2010.

9. Intrinsically Quantum-Mechanical Curvature and the Cosmological Constant Problem, seminar at Oxford University, June 2010.

10. Doing Physics with Non-Hermitian and Non-Diagonalizable Hamiltonians, seminar at Oxford University, June 2010.

11. Intrinsically Quantum-Mechanical Curvature and the Cosmological Constant Problem, seminar at Vanderbilt University, August 2010.

12. Impact of a Global Quadratic Potential on Galactic Rotation Curves, seminar at Vanderbilt University, August 2010.

13. Intrinsically Quantum-Mechanical Curvature and the Cosmological Constant Problem, invited talk presented at the International Conference on Two Cosmological Models, Universidad Iberoamericana, Mexico City, November 2010.

14. Impact of a Global Quadratic Potential on Galactic Rotation Curves, invited talk presented at the International Conference on Two Cosmological Models, Universidad Iberoamericana, Mexico City, November 2010.

15. Intrinsically Quantum-Mechanical Curvature and the Cosmological Constant Problem, seminar at McGill University, November 2010.

16. Intrinsically Quantum-Mechanical Curvature and the Cosmological Constant Problem, seminar at Universite de Montreal, November 2010.

17. Alternatives to Dark Matter, colloquium at Universite de Montreal, November 2010.

18. Impact of a Global Quadratic Potential on Galactic Rotation Curves, invited talk presented at the Miami 2010 Conference, Fort Lauderdale, December 2010.

2011

1. Observational Evidence for the Non-Diagonalizable Hamiltonian of Conformal Gravity, invited talk at the Quantum Physics with Non-Hermitian Operators Conference, Dresden, June 2011.

2. Observational Evidence for the Non-Diagonalizable Hamiltonian of Conformal Gravity, invited talk at the PT Quantum Mechanics Conference, Heidelberg, September 2011.

3. Why We Believe in Dark Matter and Dark Energy – and Do We Have To?, colloquium at Rutgers, October 2011.

4. Why We Believe in Dark Matter and Dark Energy – and Do We Have To?, colloquium at Wesleyan, November 2011.

5. The 2011 Nobel Prize in Physics, colloquium at University of Connecticut, Storrs, November 2011.

6. Cosmological Perturbations in Conformal Gravity, seminar at Universite de Montreal, November 2011.

7. Making the Case for Conformal Gravity, invited talk at the Miami 2011 Conference, Fort Lauderdale, December 2011.

2012

1. Making the Case for Conformal Gravity, seminar at Perimeter Institute, Waterloo Canada, July 2012.

2. Making the Case for Conformal Gravity, seminar at University of Utrecht, Netherlands, August 2012.

3. PT Symmetry as a Necessary and Sufficient Condition for Unitary Time Evolution, invited talk at the PHHQP XI: Non-Hermitian Operators in Quantum Physics, Paris, August 2012.

4. Why We Believe in Dark Matter and Dark Energy – and Do We Have To?, colloquium at Concordia University, Montreal, November 2012.
5. Making the Case for Conformal Gravity, seminar at Universite de Montreal, November 2012.
6. Solution to the Ghost Problem in Fourth-Order Derivative Theories and its Implications for Gravity and Astrophysics, seminar at McGill University, Montreal, November 2012.
7. PT Symmetry as a Necessary and Sufficient Condition for Unitary Time Evolution, invited talk at the Miami 2012 Conference, Fort Lauderdale, December 2012.

2013

1. Consistency of Conformal Gravity as a Microscopic Theory and its Implications for Gravity and Astrophysics as a Macroscopic One, seminar at Yale University April 2013.
2. Why there is a Cosmological Constant Problem, and what we can do about it, invited talk at Tales of Lambda Conference, Nottingham England, July 2013.
3. Why there is a Cosmological Constant Problem, and what we can do about it, seminar at Washington University, September 2013.
4. The 2013 Nobel Prize for the Higgs Boson, colloquium at the University of Connecticut, Storrs, November 2013.
5. Why there is a Cosmological Constant Problem, and what we can do about it, seminar at Syracuse University, November 2013.
6. Why Gravity cannot be Quantized Canonically, and what we can do about it, invited talk at the Miami 2013 Conference, Fort Lauderdale, December 2013.

2014

1. The Crisis in Fundamental Physics, colloquium at the University of New Haven, May 2014.
2. Introduction to PT Symmetry, invited talk at the 9th Biennial Meeting of the International Association for Relativistic Dynamics, Storrs CT, June 2014.
3. Torsion: what it is and how to constrain it, invited talk at the 9th Biennial Meeting of the International Association for Relativistic Dynamics, Storrs CT, June 2014.
4. PT Symmetry, Conformal Symmetry, and the Metrication of the Fundamental Forces, seminar at City University, London, July 2014.

5. PT Symmetry, Conformal Symmetry, and the Metrication of the Fundamental Forces, invited talk at the New England Section of the American Physical Society Meeting, Wentworth Institute of Technology, Boston, November 2014.

6. PT Symmetry, Conformal Symmetry, and the Metrication of the Fundamental Forces, seminar at the University of Connecticut, Storrs, November 2014.

7. PT Symmetry, Conformal Symmetry, and the Metrication of the Fundamental Forces, invited talk at the Discrete 2014 Conference, King's College London, December 2014.

8. PT Symmetry, Conformal Symmetry, and the Metrication of the Fundamental Forces, invited talk at the Miami 2014 Conference, Fort Lauderdale, December 2014.

2015

1. Why physicists are interested in Differential Geometry, seminar at the University of Connecticut, Storrs, February, 2015.

2. Advancing the case for PT Symmetry – the Hamiltonian is always PT Symmetric, invited talk at Quantum (and Classical) Physics with Non-Hermitian Operators (PHHQP13), Jerusalem, July 2015.

3. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, seminar at Washington University, November 2015.

4. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, invited talk at the Miami 2015 Conference, Fort Lauderdale, December 2015.

2016

1. Neutrino Oscillations – The 2015 Nobel Prize in Physics, colloquium at the University of Connecticut, Storrs, January 2016.

2. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, seminar at Imperial College London, May 2016.

3. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, seminar at King's College London, May 2016.

4. CPT Symmetry Without Hermiticity, invited talk at ICHEP2016, the 38th International Conference on High Energy Physics, Chicago, August 2016.

5. Antilinearity Rather than Hermiticity as a Guiding Principle for Quantum Theory, seminar at the University of Connecticut, Storrs, November 2016.

6. Antilinearity Rather than Hermiticity as a Guiding Principle for Quantum Theory, invited talk at the Miami 2016 Conference, Fort Lauderdale, December 2016.

2017

1. Antilinearity Rather than Hermiticity as a Guiding Principle for Quantum Theory, invited talk at Pseudo-Hermitian Hamiltonians in Quantum Physics (PHHQP17), Bad Honnef, Germany, May 2017.

2. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, seminar at Niels Bohr Institute, Copenhagen, May 2017.

3. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, seminar at the University of Liverpool, May 2017.

4. Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, seminar at the University of Manchester, May 2017.

5. The Crisis in Fundamental Physics, SUNY Albany, September 2017.

6. Quantum Conformal Gravity, seminar at University of Utrecht, Netherlands, October 2017.

7. Antilinearity Rather than Hermiticity as a Guiding Principle for Quantum Theory, seminar at Wien Technical University, Vienna, October 2017.

8. Quantum Conformal Gravity, seminar at Wien Technical University, Vienna, October 2017.

9. Is the Cosmological Constant Problem Properly Posed?, seminar at CERN, October, 2017.

10. The 2017 Nobel Prize in Physics, Colloquium at the University of Connecticut Physics Department, November, 2017.

11. The 2017 Nobel Prize in Physics, colloquium at the University of Connecticut Mathematics Department, December, 2017.

12. Anomalous Dimensions and the Renormalizability of the Four-Fermion Interaction, invited talk at the Miami 2017 Conference, Fort Lauderdale, December 2017.

2018

1. Quantum Conformal Gravity, seminar at the University of Cambridge, United Kingdom, June 7, 2018.

2. Quantum Conformal Gravity, seminar at Stanford University, Stanford Linear Accelerator Center Particle Theory Group, August 21, 2018.

3. Quantum Conformal Gravity, seminar at Chapman University Physics Department, August 30, 2018.

4. Why Physicists are Interested in Differential Geometry, seminar at Chapman University Mathematics Department, August 31, 2018.

5. Is Dark Matter Fact or Fantasy? – Clues From the Data, Stanford University, Stanford Linear Accelerator Center KAVLI Astrophysics Institute, September 7, 2018.

6. Quantum Conformal Gravity, seminar at Lawrence Berkeley Laboratory, University of California at Berkeley, September 12, 2018.

7. Quantum Conformal Gravity, seminar at University of California at Santa Cruz Center for Particle Theory, October 2, 2018.

8. Is the Cosmological Constant Problem Properly Posed?, seminar at Lawrence Berkeley Laboratory, University of California at Berkeley, October 3, 2018.

9. Quantum Conformal Gravity, seminar at Stanford University Physics Department, October 9, 2018.

10. Quantum Conformal Gravity, seminar at Washington University Physics Department, October 18, 2018.

11. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at Washington University Physics Department, October 19, 2018.

12. Quantum Conformal Gravity, seminar at Johns Hopkins University Physics Department, November 5, 2018.

13. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at NASA Goddard Space Flight Center, Greenbelt, Maryland, November 8, 2018.

14. Quantum Conformal Gravity, seminar at Ecole Polytechnique Federal de Lausanne, Lausanne, Switzerland, November 23, 2018.

15. Living Without Supersymmetry, seminar at Discrete 2018 Conference, Vienna, Austria, November 28, 2018.

16. PT and CPT Symmetry as a Dynamics, seminar at Discrete 2018 Conference, Vienna, Austria, November 29, 2018.

17. Quantum Conformal Gravity, seminar at University of Florida Physics Department, Gainesville, December 11, 2018.

18. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at Miami 2018, Fort Lauderdale, December 16, 2018.

2019

1. PT and CPT Symmetry as a Dynamics – Application to the Goldstone Theorem, seminar at King's College London, United Kingdom, January 25, 2019.

2. Making Sense of the Nambu-Jona-Lasinio Model Via Scale Invariance, seminar at Scale Invariance in Particle Physics and Cosmology Conference, CERN, Geneva, Switzerland, February 1, 2019.

3. Quantum Conformal Gravity, seminar at University of Massachusetts Physics Department, Amherst, February 22, 2019.

4. Is Dark Matter Fact or Fantasy? – Clues From the Data, colloquium presented at University of Connecticut Department of Physics, Storrs, March 1, 2019.

5. Making Sense of the Nambu-Jona-Lasinio Model Via Scale Invariance, seminar at the Stanford Linear Accelerator Center, Stanford California, July 10, 2019.

6. Making Sense of the Nambu-Jona-Lasinio Model Via Scale Invariance, seminar at the Light Cone 2019 - QCD on the light cone: from hadrons to heavy ions Conference, Paris, France, September 18, 2019.

7. Light-front Quantization is Instant-time Quantization, seminar at the Light Cone 2019 - QCD on the light cone: from hadrons to heavy ions Conference, Paris, France, September 20, 2019.

8. The 2019 Nobel Prize in Physics, colloquium at the University of Connecticut, Storrs Connecticut, October 18, 2019

9. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at University of Texas, Austin Texas, November 18, 2019.

10. Light-front Quantization is Instant-time Quantization, seminar at New Mexico State University, Las Cruces New Mexico, November 20, 2019.

11. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at New Mexico State University, Las Cruces New Mexico, November 21, 2019.

12. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at University of Saint Andrews, Saint Andrews United Kingdom, November 27, 2019.

13. Quantum Mechanics off the Beaten Track, seminar at University of Saint Andrews, Saint Andrews United Kingdom, November 28 (2019).

14. Quantum Conformal Gravity, seminar at University of Saint Andrews, Saint Andrews United Kingdom, November 29, 2019.

15. Quantum Mechanics off the Beaten Track, seminar at University of Central Florida, Orlando Florida, December 9, 2019.

16. Is Dark Matter Fact or Fantasy? – Clues From the Data, seminar at University of Central Florida, Orlando Florida, December 10, 2019.

17. Light-front Quantization is Instant-time Quantization, seminar at Miami 2019, Fort Lauderdale Florida, December 12, 2019.

2020

1. Ghost problems from Pauli-Villars to fourth-order quantum gravity and their resolution, virtual invited talk at Quantum Physics with Non-Hermitian Operators (PHHQP2020), May 21, 2020.

2. Ghost problems from Pauli-Villars to fourth-order quantum gravity and their resolution, virtual invited talk at International Association of Relativistic Dynamics Conference, June 1, 2020.

3. Ghost problems from Pauli-Villars to fourth-order quantum gravity and their resolution, virtual invited talk at Newton 1665 CERN Conference, June 29, 2020.

4. Is Dark Matter Fact or Fantasy? – Clues From the Data, virtual seminar at San Francisco State University Astronomy Group, September 20, 2020.

5. Comparing light-front quantization with instant-time quantization. virtual invited talk at the International Light Cone Advisory Committee (ILCAC) Conference, November 25, 2020.

6. Is the cosmological constant problem properly posed?, virtual Invited talk at Miami 2020, University of Miami, December 12, 2020.

2021

1. Making Sense of the Nambu-Jona-Lasinio Model Via Scale Invariance, virtual presentation at Third Proton Mass Workshop, Argonne National Laboratory, January 14, 2021.

2. Is Dark Matter Fact or Fantasy? – Clues From the Data, virtual seminar at University of Connecticut, January 27, 2021.

3. Is Dark Matter Fact or Fantasy? – Clues From the Data, virtual colloquium at San Francisco State University, February 8, 2021.

4. Quantum Mechanics off the Beaten Track, colloquium at University of Connecticut, March 5, 2021.

5. Solution to the ghost problem in higher derivative gravity, virtual presentation at Workshop on Quantum Gravity, Higher Derivatives and Nonlocality, March 8, 2021.

6. PT Symmetry, virtual seminar at Czech Technical University in Prague, Prague, Czech Republic, June 4, 2021.

7. Solution to the ghost problem in higher-derivative gravity, virtual seminar at Czech Technical University in Prague, Prague, Czech Republic, June 11, 2021.

8. Solution to the ghost problem in higher-derivative gravity, virtual seminar at University of Marseille, France, June 18, 2021.

9. Making Sense of the Nambu-Jona-Lasinio Model Via Scale Invariance, virtual seminar at University of Marseille, France, July 2, 2021.

10. Extension of the Goldstone and the Englert-Brout-Higgs mechanisms to non-Hermitian theories, virtual invited talk at the 15th Analytic and Algebraic Methods in Physics Conference, Prague, Czech Republic, September 3, 2021.

11. Extension of the Goldstone and the Englert-Brout-Higgs mechanisms to non-Hermitian theories, virtual invited talk at Quantum Physics with Non-Hermitian Operators (PHHQP2020), September 9, 2021.

12. Light-Front Quantization From Then Until Now, vitual invited talk at the Light Cone 2021 - Physics of Hadrons on the Light Front Conference, Jeju Island, Korea, November 30, 2021.

13. Critique of the Use of Geodesics in Astrophysics and Cosmology, virtual Invited talk at Miami 2021, University of Miami, December 16, 2021.

2022

1. Solution to the ghost problem in higher-derivative gravity, virtual seminar at King's College London, United Kingdom, January 12, 2022.

2. How to quantize gravity and how not to quantize gravity, vitual presentation at Quantum Gravity, Cosmology and Black Holes Conference, March 17, 2022.

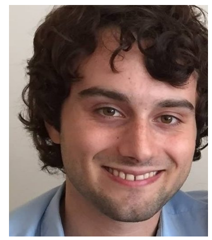
3. How to quantize gravity and how not to quantize gravity, vitual presentation at General Relativity, Quantum Mechanics and Everything in Between, April 25, 2022.

Yaobing Deng

James O'Brien

Matthew Phelps

Asanka Amarasinghe



Daniel Norman

Tianye Liu

Ionel Simbotin

David Cox

Joe Poveromo

Jan Kmetko

Students who worked with me

General Relativity and Gravitation, Vol. 20, No. 10, 1988

Perfect Maxwell Fluids in the Standard Cosmology

Yaobing Deng[1] and Philip D. Mannheim[1]

Received March 27, 1987

We provide closed form solutions to the Maxwell equations for the modes associated with the propagation of classical electromagnetic radiation in background Robertson–Walker geometries of any allowed spatial curvature. An incoherent averaging over complete sets of these modes in each spatial geometry is then found to yield the familiar perfect fluid energy-momentum tensor required of the standard Friedmann cosmology.

1. INTRODUCTION

In the standard Friedmann cosmology the energy-momentum tensor is taken, usually without too much discussion, to be of the perfect fluid form. Now in the matter-dominated era of the standard cosmology, the gravitational source is composed of noninteracting pressureless dust particles whose dynamics is fixed entirely by the Einstein equations alone, thereby forcing the particles to move on the geodesics appropriate to the geometry under consideration and automatically enforcing the energy-momentum tensor to be of the desired perfect fluid form. The same cannot immediately be said, however, of the radiation-dominated era, since there the energy-momentum tensor is composed of waves which fill all space and are not restricted (except in the geometrical optics limit) to move on geodesics. Moreover, the equations of motion of these waves as they propagate in a given geometry are independent of and not fixed by the Einstein equations. Thus the establishing of a perfect fluid form for the energy-momentum tensor in the radiation-dominated era requires a more detailed calculation. A recent paper [1] shows how the familiar perfect fluid energy-momentum tensor does in fact arise in a simple model in

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842/20/10-1

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SELF-CONSISTENT SOLUTION FOR A SCALAR FIELD COUPLED CONFORMALLY TO A ROBERTSON-WALKER GEOMETRY

YAOBING DENG AND PHILIP D. MANNHEIM
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Received 1987 February 17; accepted 1987 June 23

ABSTRACT

We present the general exact nonstatic solutions for a conformally invariant massless scalar field coupled to a Robertson-Walker geometry. Solutions are provided for all the three appropriate spatial geometries associated with the metric. The purely time-dependent, spatially independent solutions for the scalar field that we find play the same role for the Robertson-Walker cosmology as a pure spacetime independent field does for the inflationary de Sitter cosmology.

Subject heading: cosmology

The general covariant action of a massless scalar field $S(x)$ coupled conformally to gravity is given by

$$I = \int (-g)^{1/2}\left[\left(\frac{1}{16\pi G} - \frac{S^2}{12}\right)R^\alpha{}_\alpha + \frac{1}{2} S_\alpha S^\alpha\right]d^4x \, . \tag{1}$$

Variation of this action with respect to $g_{\mu\nu}(x)$ yields the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu} R^\alpha{}_\alpha = -8\pi G T_{\mu\nu} \tag{2}$$

and serves to define a generally covariant energy-momentum tensor

$$T_{\mu\nu} = \tfrac{2}{3}S_\mu S_\nu - \tfrac{1}{6}g_{\mu\nu} S_\alpha S^\alpha - \tfrac{1}{3}SS_{\mu;\nu} + \tfrac{1}{3}g_{\mu\nu} SS^\alpha{}_{;\alpha} - \tfrac{1}{6}S^2(R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu} R^\alpha{}_\alpha) \, . \tag{3}$$

Similarly, variation of the action with respect to $S(x)$ yields the scalar field equation of motion

$$(-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu S] = -\tfrac{1}{6}SR^\alpha{}_\alpha \, . \tag{4}$$

With the use of this equation of motion it then follows that $T_{\mu\nu}$ is covariantly traceless as is to be expected in a conformal invariant field theory. From the Einstein equations it further follows that $R_{\mu\nu}$ is also traceless so that the Ricci scalar vanishes. In this paper we shall seek self-consistent solutions to the above set of coupled equations in the case where the geometry is to be described by a Robertson-Walker metric with line element

$$g_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + R^2(t)\left[\frac{dr^2}{(1 - Kr^2)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] . \tag{5}$$

Since the Robertson-Walker metric is maximally symmetric in the three space associated with the three ordinary spatial coordinates, the Einstein equations then oblige the energy-momentum tensor to possess this same very high symmetry and thus take the form of a perfect fluid. Thus if solutions to the coupled field equations exist at all we anticipate that there should be one in which the scalar field solution also possesses this symmetry and thus only depends on time and not on the spatial coordinates at all.[1] Consequently equation (4) must reduce to

$$\partial_t[R^3(t)\dot{S}(t)] = 0 \, , \tag{6}$$

so that

$$\dot{S}(t) = \frac{C}{R^3(t)} \, , \tag{7}$$

where C is an integration constant. For the Robertson-Walker metric a specific form for $R(t)$ can be found directly since the vanishing of the Ricci scalar entails that

$$R(t)\ddot{R}(t) + \dot{R}^2(t) + K = 0 \tag{8}$$

and yields

$$R^2(t) = A + Bt - Kt^2 \, , \tag{9}$$

[1] The procedure that we shall use is self-consistent in that we assume a geometry, solve for the scalar field propagating in that geometry, and then feed the scalar field solution into the Einstein equations to show that it then yields the assumed geometry back again. Thus while our approach shows that we have solutions it does

BLACK-BODY RADIATION IN A CURVED ROBERTSON–WALKER BACKGROUND

YAOBING DENG and PHILIP D. MANNHEIM

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(Received 16 February, 1987)

Abstract. In the standard Friedmann cosmology the black-body radiation spectrum is usually taken (without explicit proof as far as we know) to have the same familiar T^4-form that it has in a flat space. With explicit use of the equation of motion of a quantized massless field propagating in a curved background Robertson–Walker metric we show (for the readily tractable scalar field case) that the assumption is in fact true for an open Universe. For a closed Universe, we find that there is an in principle modification to the T^4-law. Unfortunately, the correction turns out to be too small to be experimentally detectable. In passing, we also obtain a simple derivation for the cosmological red shift of frequencies.

1. Introduction

In the standard treatment of the Friedmann cosmology the energy-momentum tensor is usually taken to be a perfect fluid whose radiation component behaves like a perfect black body with its familiar T^4 power spectrum. Now, while such a form for the black-body power-spectrum is readily establishable in flat space we are not aware of any explicit derivation in the literature of a form for the black-body spectrum in a curved one. In this paper we shall, therefore, study the black-body power spectrum in a curved space by explicitly quantizing a massless scalar field (the most tractable case, and one which is rich enough to indicate the general procedure) propagating in a curved Robertson–Walker (R.W.) background metric. With the explicit use of the scalar field equation of motion (i.e., the equation of motion of the source of the gravitational field) we find that for an open Universe (i.e., one with spatial curvature K either 0 or -1) the black-body spectrum is in fact the flat space one. However, in a closed ($K = +1$) Universe we find that the spectrum is actually slightly modified. Unfortunately, the correction turns out to be too small to be observable experimentally, so that it does not appear that experimental study of this aspect of the cosmic microwave background radiation spectrum will currently enable us to distinguish between an open and a closed Universe. In a previous paper (Mannheim and Kazanas, 1985) the purely classical coupled system of gravity and radiation was studied and it was shown there how the standard perfect fluid form for the energy-momentum tensor is actually obtained in the first place. In this paper we continue that study by looking at classical gravity coupled this time to quantized radiation.

The present paper is organized as follows. In Section 2 we present the solutions to the equation of motion of a classical massless scalar field coupled to a background R.W. metric, and in Section 3 we canonically quantize the scalar field in the same curved background. Finally, in Section 4 we study the thermodynamics of a gas of the quantum

PHYSICAL REVIEW D VOLUME 44, NUMBER 6 15 SEPTEMBER 1991

Acceleration-free spherically symmetric inhomogeneous cosmological model with shear viscosity

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(Received 17 August 1990)

Some new exact solutions to the Einstein equations with an acceleration-free imperfect-fluid source are obtained. Some physical restrictions on the solutions are discussed. Cosmological models built out of these solutions are found to have increasing entropy per baryon and not possess any flatness problem.

I. INTRODUCTION

This is the second in a series of two papers in which we explore imperfect-fluid cosmological models and obtain some new exact solutions to them. The models that we study have the feature that while the underlying associated geometry is in general an inhomogeneous one, the models nonetheless evolve so that at late times the inhomogeneities die out and the Universe becomes the familiar highly symmetric Friedmann-Robertson-Walker one of today. To a present-day observer our models are indistinguishable from the standard one and must hence be regarded as observationally viable. However, since each of our models has a history which is different from that of the standard model, our models do not all suffer from some of the familiar difficulties (horizon, entropy, and flatness problems) which the standard model possesses. They thus provide for potentially interesting cosmologies. In our accompanying companion paper [1] we study shear-free cosmological models with heat flow and bulk viscosity and in the present paper we study acceleration-free ones with shear viscosity. In all the cases the specific non-perfect-fluid terms are found to lead to some interesting implications for cosmology. For a complete discussion of our motivation, and for our formulation of the problem and its notation we refer the reader to Ref. [1].

In this particular paper we study Einstein's field equations with an acceleration-free imperfect-fluid source with shear viscosity coefficient $\eta(r,t)$ and associated energy-momentum tensor [$\rho(r,t)$ and $p(r,t)$ are the standard energy density and pressure of the fluid]

$$T_{\mu\nu}=\rho U_\mu U_\nu+pH_{\mu\nu}-2\eta\sigma_{\mu\nu}\ , \tag{1.1}$$

where

$$H_{\mu\nu}=g_{\mu\nu}+U_\mu U_\nu\ , \tag{1.2}$$

$$2\sigma_{\mu\nu}=H_\mu{}^\alpha H_\nu{}^\beta(U_{\alpha;\beta}+U_{\beta;\alpha}-\tfrac{2}{3}g_{\alpha\beta}U^\gamma{}_{;\gamma})\ , \tag{1.3}$$

and where U_μ is the four-velocity of the fluid. We take the geometry to be spherically symmetric about a single point, and thus isotropic but not homogeneous at arbitrary times. Further, we take the geometry to be acceleration-free (viz. $U^\alpha U_{\beta;\alpha}=0$), so that the most general admissible metric then takes the form

$$ds^2=-dt^2+e^{2\lambda}dr^2+Y^2(d\theta^2+\sin^2\theta\, d\phi^2)\ , \tag{1.4}$$

where λ and Y are functions of r and t only; while the fluid four-velocity vector itself then simplifies to

$$U_\mu=(-1,0,0,0) \tag{1.5}$$

so that the fluid is comoving with the geometry. In this geometry the Einstein equations take the form

$$\kappa\rho=\frac{1}{Y^2}+2\dot\lambda\frac{\dot Y}{Y}+\frac{\dot Y^2}{Y^2}-e^{-2\lambda}\left[2\frac{Y''}{Y}-2\lambda'\frac{Y'}{Y}+\frac{Y'^2}{Y^2}\right]\ , \tag{1.6}$$

$$\kappa\left[p-\frac{4}{3}\eta\left[\dot\lambda-\frac{\dot Y}{Y}\right]\right]=-\frac{1}{Y^2}-2\frac{\ddot Y}{Y}-\frac{\dot Y^2}{Y^2}+e^{-2\lambda}\frac{Y'^2}{Y^2}\ , \tag{1.7}$$

$$\kappa\left[p+\frac{2}{3}\eta\left[\dot\lambda-\frac{\dot Y}{Y}\right]\right]=-\ddot\lambda-\dot\lambda^2-\dot\lambda\frac{\dot Y}{Y}-\frac{\ddot Y}{Y}+e^{-2\lambda}\left[\frac{Y''}{Y}-\lambda'\frac{Y'}{Y}\right]\ , \tag{1.8}$$

$$\frac{\dot Y'}{Y}-\dot\lambda\frac{Y'}{Y}=0\ , \tag{1.9}$$

where κ denotes the quantity $8\pi G$, so that the Bianchi identities impose the following two constraints on the fluid:

$$\dot\rho+\left[\dot\lambda+2\frac{\dot Y}{Y}\right](\rho+p)=\frac{4}{3}\eta\left[\dot\lambda-\frac{\dot Y}{Y}\right]^2 \tag{1.10}$$

and

$$p'=\frac{4}{3}\frac{\partial}{\partial r}\left[\eta\left[\dot\lambda-\frac{\dot Y}{Y}\right]\right]+4\eta\left[\dot\lambda-\frac{\dot Y}{Y}\right]\frac{Y'}{Y}\ . \tag{1.11}$$

(In passing we note that because of the conservation of the energy-momentum tensor we find that unlike the perfect-fluid case where some acceleration is necessary to support a pressure gradient, for an imperfect fluid this gradient may be supported by the viscosity instead.)

In attempting to find solutions to the Einstein equations we note first that the integration of Eq. (1.9) is straightforward, yielding

PHYSICAL REVIEW D VOLUME 42, NUMBER 2 15 JULY 1990

Shear-free spherically symmetric inhomogeneous cosmological model with heat flow and bulk viscosity

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An exact solution to the Einstein equations with a shear-free imperfect-fluid source is obtained. The solution approaches a locally flat Robertson-Walker one in the large-t limit and thus serves as a viable candidate for a realistic cosmological model. The model built out of this solution is found to be free of horizon, entropy, and flatness problems.

I. INTRODUCTION

The standard Friedmann-Robertson-Walker big-bang model provides a very attractive framework for discussing the large-scale cosmology of the Universe. Nonetheless, the model has some major drawbacks, the most prominent being the horizon, flatness, and entropy problems. These problems arise mainly because of an insistence that the Universe is not only currently in a Robertson-Walker phase but that it had also been in one for almost all of its entire previous history as well. Moreover, this point of view is even reinforced by the very ability of the popular inflationary universe model[1] to provide a candidate solution to all of the standard model problems as a consequence of very early Universe dynamics which occurred even prior to the onset of the conventional radiation-dominated Robertson-Walker phase.

Now though the rationale for and the achievements of the inflationary universe model are, of course, well known (a recent review of its current status may be found in Ref. 1), it is nonetheless useful to recall here some specific features of the familiar standard model problems that inflation potentially resolves. The horizon problem is a problem of the Robertson-Walker geometry itself since once we are given the current degree of isotropy of the microwave background it follows that an earlier time Robertson-Walker universe could not have been causally connected. The problem of understanding the huge entropy of the current Universe is due to the dynamical assumption that the energy-momentum tensor is and always has been that of an adiabatic and hence entropy-conserving perfect fluid. Finally, the flatness problem is a problem which arises because of the structure of the Einstein equations themselves in the standard cosmology, since it requires extreme fine-tuning to have the current energy density of the Universe be so close to the critical energy density for a Universe as old as ours appears to be. This flatness problem is thus a far more detailed dynamical problem than the horizon and entropy problems, which themselves are essentially kinematical in nature. The three standard problems are thus somewhat independent of each other and to seek a resolution of all three problems will thus require a relaxation of quite a few of the standard-model assumptions. As we will see, the imperfect-fluid model that we consider here will exactly do this for us.

To motivate our study we note that while we observe the current Universe to be one which is expanding and which has a matter distribution which is maximally three-space symmetric on the largest scales to good accuracy (this latter feature is actually still open to question), that does not mean that the Universe has always been that way. Nonetheless, it is reasonable to at least assume that the Universe has always been expanding so that there actually was a hot early Universe. However, the question of whether or not it has always been maximally three-space symmetric is not one that we can as readily respond to. Consequently, we can entertain the possibility that the Universe may not always have been as highly symmetric spatially as we now observe it to be, but rather that it only evolved over the course of time into its present highly symmetric form, and then perhaps only quite slowly. In such a situation the horizon, flatness, and entropy problems would all have to be reexamined anew.

In order to try to determine what lower symmetry and what specific dynamics the not-so-recent Universe could possibly have possessed, we shall, in this paper, take into consideration some of the implications of kinetic theory for the fluid dynamics of the Universe. To motivate our approach here we recall some of the characteristic features of standard non-relativistic classical kinetic theory. For our purposes the most significant aspect of Boltzmann transport theory is that the equilibrium Maxwell-Boltzmann distribution, a distribution which is itself independent of the specific collisional forces associated with the system of interest, will be the infinite-time solution to the Boltzmann transport equation provided that there actually are collisions in the first place. Thus while collisions are necessary to thermalize the system, the distribution that is eventually produced is in fact independent of the collisions which produce it. Moreover, at finite times, the system would necessarily not be in a Maxwell-Boltzmann distribution (this distribution not being a finite-time solution to the Boltzmann equation), so

Linear Potentials and Galactic Rotation Curves - Detailed Fitting *

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Abstract

We continue our study of the astrophysical implications of the linear potential $V(r) = -\beta c^2/r + \gamma c^2 r/2$ associated with fundamental gravitational sources in the conformal invariant fourth order theory of gravity which has recently been advanced by Mannheim and Kazanas as a candidate alternative to the standard second order Einstein theory. We provide fitting to the rotation curves of an extensive and diverse set of 11 spiral galaxies whose data are regarded as being particularly reliable. Without the assumption of the existence of any dark matter the model is found to fit the shapes of the rotation curves extremely well, but with a pattern of normalizations which proves to be very instructive.

1 Introduction

During the last few years Mannheim and Kazanas (Mannheim 1990, 1992, 1993a, b, 1994, 1995a, b, c; Mannheim and Kazanas 1989, 1991, 1994; Kazanas and Mannheim 1991) have been exploring conformal gravity (viz. gravity based on invariance of the geometry under any and all local conformal stretchings of the form $g_{\mu\nu}(x) \rightarrow \Omega(x) g_{\mu\nu}(x)$) as a covariant candidate alternative to the standard Newton-Einstein gravitational theory. Their study has entailed both the examining of the formal structure of the theory and the identification of its possible observational astrophysical implications. In particular they found (Mannheim and Kazanas 1989; see also Riegert 1984) the most general, exact all order metric exterior to a static, spherically symmetric source such as a star in the theory, viz. (in standard static coordinates)

$$-g_{00} = 1/g_{rr} = 1 - \beta(2 - 3\beta\gamma)/r - 3\beta\gamma + \gamma r - kr^2 \qquad (1)$$

where β, γ and k are three integration constants. Subsequently, they also found (Mannheim and Kazanas 1994) the associated exact interior solution and established its consistency

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Completeness of non-normalizable modes

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Abstract

We establish the completeness of some characteristic sets of non-normalizable modes by constructing fully localized square steps out of them, with each such construction expressly displaying the Gibbs phenomenon associated with trying to use a complete basis of modes to fit functions with discontinuous edges. As well as being of interest in and of itself, our study is also of interest to the recently introduced large extra dimension brane-localized gravity program of Randall and Sundrum, since the particular non-normalizable mode bases that we consider (specifically the irregular Bessel functions and the associated Legendre functions of the second kind) are associated with the tensor gravitational fluctuations which occur in those specific brane worlds in which the embedding of a maximally four-symmetric brane in a five-dimensional anti-de Sitter bulk leads to a warp factor which is divergent. Since the brane-world massless four-dimensional graviton has a divergent wavefunction in these particular cases, its resulting lack of normalizability is thus not seen to be any impediment to its belonging to a complete basis of modes, and consequently its lack of normalizability should not be seen as a criterion for not including it in the spectrum of observable modes. Moreover, because the divergent modes we consider form complete bases, we can even construct propagators out of them in which these modes appear as poles with residues which are expressly finite. Thus, even though normalizable modes appear in propagators with residues which are given as their finite normalization constants, non-normalizable modes can just as equally appear in propagators with finite residues too—it is just that such residues will not be associated with bilinear integrals of the modes.

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1. Introduction

In constructing complete bases of mode solutions to wave equations it is very convenient to work with modes which are normalizable since they obey a closure relation. Specifically, if

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RESEARCH ARTICLE

Limitations of the standard gravitational perfect fluid paradigm

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Abstract We show that the standard perfect fluid paradigm is not necessarily a valid description of a curved space steady state gravitational source. Simply by virtue of not being flat, curved space geometries have to possess intrinsic length scales, and such length scales can affect the fluid structure. For modes of wavelength of order or greater than such scales eikonalized geometrical optics cannot apply and rays are not geodesic. A set of wave mode rays that would all be geodesic in flat space (where there are no intrinsic length scales) and form a flat space perfect fluid would not all remain geodesic or of the perfect fluid form when the system is covariantized to curved space. Covariantizing thus entails not only the replacing of flat space functions by covariant ones, but also the introduction of intrinsic scales that were absent in flat space. In principle it is thus unreliable to construct the curved space energy–momentum tensor as the covariant generalization of a geodesic-based flat spacetime energy–momentum tensor. By constructing the partition function as an incoherent average over a complete set of modes of a scalar field propagating in a curved space background, we show that for the specific case of a static, spherically symmetric geometry, the steady state energy–momentum tensor that ensues will in general be of the form $T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu} + \pi_{\mu\nu}$ where the anisotropic $\pi_{\mu\nu}$ is a symmetric, traceless rank two tensor which obeys $U^\mu \pi_{\mu\nu} = 0$. Such a $\pi_{\mu\nu}$ type term is absent for an incoherently averaged steady state fluid in a spacetime where there are no intrinsic length scales, and in principle would thus be missed in a covariantizing of a flat

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Impact of a Global Quadratic Potential on Galactic Rotation Curves

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We present a conformal gravity fit to the 20 largest of a sample of 110 spiral galaxies. We identify the presence of a universal quadratic potential $V_{\kappa}(r) = -\kappa c^2 r^2/2$ with $\kappa = 9.54 \times 10^{-54}\text{cm}^{-2}$ induced by cosmic inhomogeneities. When $V_{\kappa}(r)$ is taken in conjunction with both a universal linear potential $V_{\gamma_0}(r) = \gamma_0 c^2 r/2$ with $\gamma_0 = 3.06 \times 10^{-30}\text{cm}^{-1}$ generated by the homogeneous cosmic background and the contribution generated by the local luminous matter in galaxies, the theory then accounts for the rotation curve systematics observed in the entire 110 galaxies, without the need for any dark matter whatsoever. Our study suggests that using dark matter may be nothing more than an attempt to describe global effects in purely local galactic terms. With $V_{\kappa}(r)$ being negative, galaxies can only support bound orbits up to distances of order $\gamma_0/\kappa = 100\text{kpc}$, with global physics imposing a limit on the size of galaxies.

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I. Introduction.—At the present time it is widely believed that on scales much larger than solar-system-sized ones astrophysical and cosmological phenomena are controlled by dark matter and dark energy, with luminous matter being only a minor contributor. However, given the lack to date of either direct detection of dark matter particles or of a solution to the cosmological constant problem, a few authors (see, e.g., [1] for a recent review) have ventured to suggest that the standard dark matter and dark energy picture may be incorrect, and that one instead needs to modify the standard Newton-Einstein gravitational theory that leads to that picture in the first place. In this Letter we study one specific alternative to Einstein gravity that has been advanced, namely, conformal gravity. We report here on the results of a conformal gravity study of the instructive 20 largest of a full sample of 110 galaxies, all of whose rotation curves we have been able to fit without the need for any dark matter whatsoever.

In seeking an alternative to Einstein gravity that is to address both the dark matter and dark energy problems, our strategy is to seek some alternate, equally metric-based theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of Einstein gravity, that yields a geometry that is described by the Ricci-flat Schwarzschild metric on solar-system-sized distance scales while departing from it on larger scales where the dark matter problem is first encountered, and that has a symmetry that can control the cosmological constant Λ. All of these criteria are met in the conformal gravity theory (see, e.g., [1]) that was first developed by Weyl. Specifically, as well as coordinate invariance, in addition one requires that the action be left invariant under local conformal transformations of the form $g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)} g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Given this requirement, the gravitational action is then uniquely prescribed to be of the form $I_W = -2\alpha_g \int d^4x(-g)^{1/2}[R_{\mu\kappa}R^{\mu\kappa} - (1/3) \times (R^{\alpha}{}_{\alpha})^2]$ where α_g is a dimensionless gravitational coupling constant. With the conformal symmetry forbidding the presence of any fundamental Λ term in I_W, conformal gravity has a control on Λ that is not possessed by Einstein gravity, and through this control conformal gravity is then able to solve the cosmological constant problem [2]. In addition, the conformal gravity equations of motion are given by [1]

$$4\alpha_g W^{\mu\nu} = T^{\mu\nu} \tag{1}$$

where $W^{\mu\nu}$ is a derivative function of $R^{\mu\nu}$. With $W^{\mu\nu}$ vanishing when $R^{\mu\nu}$ vanishes [1], Schwarzschild is thus a vacuum solution to conformal gravity, just as required [3].

II. Universal potentials from the rest of the Universe.—Since $W^{\mu\nu}$ is a derivative function of $R^{\mu\nu}$, it could potentially vanish even if the geometry is not Ricci flat, and the conformal theory could thus have non-Schwarzschild vacuum solutions as well. To identify such solutions, Mannheim and Kazanas solved for the metric associated with a static, spherically symmetric source, to find [4] that due to the underlying conformal symmetry one could bring the exact, all-order line element to the form $ds^2 = -B(r)dt^2 + dr^2/B(r) + r^2 d\Omega_2$. And with $3(W^0{}_0 - W^r{}_r)/B(r)$ then evaluating to $\nabla^4 B(r)$, the metric coefficient $B(r)$ is found to obey the remarkably simple and exact fourth-order derivative equation

$$\nabla^4 B(r) = f(r) \tag{2}$$

where $f(r) = 3(T^0{}_0 - T^r{}_r)/4\alpha_g B(r)$. For a local source of radius r_0 embedded in an empty vacuum (2) possesses an exterior solution of the form

$$B(r > r_0) = 1 - 2\beta/r + \gamma r. \tag{3}$$

Through the γr term the conformal gravity metric thus departs from the exterior Schwarzschild metric at large r alone, just as we want.

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Fitting dwarf galaxy rotation curves with conformal gravity

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ABSTRACT

We continue our study of the application of the conformal gravity theory to galactic rotation curves. Previously we had studied a varied 111 spiral galaxy sample consisting of high surface brightness galaxies, low surface brightness galaxies and dwarf galaxies. With no free parameters other than galactic mass-to-light ratios, we had found that the theory is able to account for the systematics that are observed in the entire set of galactic rotation curves without the need for any dark matter whatsoever. In this paper, we extend our study to an additional set of 27 galaxies of which 25 are dwarf galaxies, and provide updated studies of three additional galaxies that had been in the original sample, and again without dark matter find fully acceptable fits, save only for just a few galaxies that we find to be somewhat troublesome. Our current study brings to 138 the number of rotation curves of galaxies that have been accounted for by the conformal gravity theory. Since one of the primary ingredients in the theory is a universal contribution to galactic motions coming from matter exterior to the galaxies, and thus independent of them, our study reinforces one of the central concepts of the conformal gravity studies, namely that invoking dark matter should be viewed as being nothing more than an attempt to describe global physics contributions in purely local galactic terms.

Key words: galaxies: dwarf – galaxies: fundamental parameters – galaxies: general – galaxies: kinematics and dynamics.

1 INTRODUCTION

As a possible alternative to standard Einstein gravity, Weyl introduced conformal gravity in the very early days of general relativity. It is an attractive theory in that it is a pure metric theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of standard gravity while augmenting it with an additional symmetry, local conformal invariance, in which the action is left invariant under local conformal transformations on the metric of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Under such a symmetry, the gravitational action is uniquely prescribed to be of the form (see e.g. Mannheim 2006)

$$I_{\rm W} = -\alpha_{\rm g} \int d^4x\,(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$$
$$\equiv -2\alpha_{\rm g} \int d^4x\,(-g)^{1/2} \left[R_{\mu\kappa} R^{\mu\kappa} - (1/3)(R^{\alpha}{}_{\alpha})^2 \right], \qquad (1)$$

where

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}\left(g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}\right)$$
$$+ \frac{1}{6} R^{\alpha}{}_{\alpha} \left(g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}\right) \qquad (2)$$

is the conformal Weyl tensor and the gravitational coupling constant $\alpha_{\rm g}$ is dimensionless. With the conformal symmetry forbidding the presence of any $\int d^4x(-g)^{1/2}\Lambda$ term in the action, the conformal theory has a control over the cosmological constant that the standard Einstein theory does not, and through this control one is able to both address and resolve the cosmological constant problem (Mannheim 2011a). Similarly, with the coupling constant $\alpha_{\rm g}$ being dimensionless, unlike standard gravity conformal gravity is renormalizable, and with it having been shown (Bender & Mannheim 2008a,b; Mannheim 2011a) to be unitary at the quantum level, the theory is offered (Mannheim 2011b) as a consistent theory of quantum gravity in four space–time dimensions.

With the conformal theory being a consistent, renormalizable quantum theory at the microscopic level, then just as with electrodynamics, one is assured that its macroscopic classical predictions are reliable and will not be ruined by quantum corrections. Consequently, application of the theory to astrophysical phenomena allows one to test the theory. Early work in this direction was provided in Mannheim (1997) where the theory was used to fit the rotation curves of a set of 11 spiral galaxies, with the mass-to-light ratio (*M*/*L*) of the luminous optical disc of each galaxy being the only free parameters, and with no dark matter being required. More recently, Mannheim & O'Brien (2010, 2011) conducted a systematic, broad-based study of the rotation curves of a varied set of 111 galaxies (consisting of high surface brightness galaxies, low

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Galactic rotation curves in conformal gravity

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Abstract. We review some recent work by Mannheim and O'Brien on the systematics of galactic rotation curves in the conformal gravity theory. In this work the conformal theory was applied to a comprehensive, high quality sample of spiral galaxies whose rotation curves extend well beyond the galactic optical disks. On galactic scales the conformal gravitational theory departs from the standard Newtonian theory in two distinct ways. One is a local way in which local matter sources within galaxies generate not just Newtonian potentials but linear potentials as well. The other is a global way in which two universal global potentials, one linear the other quadratic, are generated by the rest of the matter in the universe. The study involves a broad set of 138 spiral galaxies of differing luminosities and sizes, and is augmented here through the inclusion of an additional three tidal dwarf galaxies. With its linear and quadratic potentials the conformal theory can account for the systematics of an entire 141 galaxy sample without any need for galactic dark matter, doing so with only one free parameter per galaxy, namely the visible galactic mass to light ratio.

1. Introduction

Over the past three decades, the persistence of the missing mass or dark matter problem has generated an increasing interest in alternative gravitational theories. In the time since the classic studies (see e.g. [1]) of the missing mass problem in spiral galaxies galactic observational techniques have improved, and it has become possible to study and constrain the motions of luminous matter in galaxies with great precision. With rotational data for spiral galaxies now extending well beyond galactic optical disks, one finds that in essentially every case studied the measured rotational velocities do not conform with the familiar Newtonian gravity expectation associated with the observed visible material. It is important to note that the Newtonian expectation on galactic distance scales is derived by extrapolating standard gravity beyond solar system distance scales without any modification. To determine the expected Newtonian prediction one treats the galaxy as a collection of N^* individual sources each of typical mass $M_\odot$ and combines the Newtonian gravitational potentials generated by each of the individual sources. For a thin disk-shaped galaxy with a typical exponential surface brightness distribution $\Sigma(R) = \Sigma_0 e^{-R/R_0}$ with scale length R_0, the resulting circular velocity for a test particle at a radial distance R from the center of the disk is given by the Freeman formula (see e.g. [2])

$$v^2(R) = \frac{N^* M_\odot G R^2}{2R_0^3}\left[I_0\left(\frac{R}{2R_0}\right)K_0\left(\frac{R}{2R_0}\right) - I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right)\right]. \qquad (1)$$

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RESEARCH ARTICLE

Gravitational analog of Faraday's law via torsion and a metric with an antisymmetric part

Philip D. Mannheim · J. J. Poveromo

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Abstract In this paper we show that in the presence of torsion and a metric with an antisymmetric part one can construct a gravitational analog of Faraday's law of electromagnetism.

Keywords Gravitational Faraday Law · Torsion · Non-symmetric gravity

1 Introduction

In a curved Riemannian background the covariant equations of motion for the electromagnetic field $F_{\mu\nu}$ take the form

$$\nabla_\nu F^{\nu\mu} = J^\mu, \tag{1}$$
$$(-g)^{-1/2}\epsilon^{\mu\nu\sigma\tau}\nabla_\nu F_{\sigma\tau} = 0, \tag{2}$$

where g is the determinant of the metric $g_{\mu\nu}$. Equation (2) can also be written in the convenient form

$$\nabla_\nu F_{\sigma\tau} + \nabla_\tau F_{\nu\sigma} + \nabla_\sigma F_{\tau\nu} = 0, \tag{3}$$

and for brevity, we shall refer to Eq. (2) as Faraday's Law even as it encompass Gauss' Law of Magnetism as well. Because of the antisymmetry of the $\epsilon^{\mu\nu\sigma\tau}$ tensor density

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Fitting galactic rotation curves with conformal gravity and a global quadratic potential

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We apply the conformal gravity theory to a sample of 111 spiral galaxies whose rotation curve data points extend well beyond the optical disk. With no free parameters other than galactic mass-to-light ratios, the theory is able to account for the systematics that is observed in this entire set of rotation curves without the need for any dark matter at all. In previous applications of the theory, a central role was played by a universal linear potential term $V(r) = \gamma_0 c^2 r/2$ that is generated through the effect of cosmology on individual galaxies, with the coefficient $\gamma_0 = 3.06 \times 10^{-30}\ \mathrm{cm}^{-1}$ being of cosmological magnitude. Because the current sample is so big and encompasses some specific galaxies whose data points go out to quite substantial distances from galactic centers, we are able to identify an additional globally induced universal term in the data, a quadratic $V(r) = -\kappa c^2 r^2/2$ term that is induced by inhomogeneities in the cosmic background. With κ being found to be of magnitude $\kappa = 9.54 \times 10^{-54}\ \mathrm{cm}^{-2}$, through study of the motions of particles contained within galaxies we are thus able to both detect the presence of a global de Sitter-like component and provide a specific value for its strength. Our study suggests that invoking dark matter may be nothing more than an attempt to describe global physics effects such as these in purely local galactic terms.

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I. INTRODUCTION

Observational studies of spiral galaxies have repeatedly established that galactic rotational velocities look nothing like the velocities that would be produced by the Newtonian gravitational potentials associated with the luminous matter in the galaxies. In consequence, it is quite widely thought that such velocity discrepancies are to be explained by the presence of copious amounts of nonluminous or dark matter in galaxies. Since the case for the presence of such dark matter rests solely on the assumption that wisdom acquired from studies on solar system distance scales can be extrapolated without modification to the much larger galactic distance scales, a few authors have ventured to suggest (see e.g. [1] for a recent review) that dark matter may not actually exist and that instead it is the standard Newtonian description that needs modifying. In this work, we apply one particular candidate alternative theory, namely, conformal gravity, to a large and comprehensive sample of 111 galactic rotation curves. With only one free parameter per galaxy, the galactic mass-to-light ratio, we find that the conformal theory provides for a good accounting of the data without the need for any dark matter at all. Moreover, because our sample is so large, through our fitting we are able to find evidence in the data for the presence of a universal quadratic potential term that the conformal theory possesses.

As a theory, conformal gravity (see e.g. [1]) is a completely covariant metric theory of gravity that possesses all the general coordinate invariance and equivalence principle structure of standard Einstein gravity, but which in addition possesses a local conformal invariance in which the action is left invariant under local metric transformations of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x)$ with any arbitrary local phase $\alpha(x)$. As a symmetry, conformal invariance forbids the presence of any fundamental cosmological constant term in the gravitational action, with the action being uniquely prescribed by the Weyl (W) action

$$I_{\mathrm{W}} = -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \equiv -2\alpha_g \int d^4x(-g)^{1/2} [R_{\mu\kappa}R^{\mu\kappa} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2], \quad (1)$$

where

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \tfrac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \tfrac{1}{6}R^{\alpha}{}_{\alpha}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}) \quad (2)$$

is the conformal Weyl tensor and the gravitational coupling constant α_g is dimensionless. Thus, unlike the standard Einstein-Hilbert (EH) action $I_{\mathrm{EH}} = -(1/16\pi G) \times \int d^4x(-g)^{1/2} R^{\alpha}{}_{\alpha}$, which can be augmented to include a $\int d^4x(-g)^{1/2}\Lambda$ term, the conformal theory has a control over the cosmological constant that the standard Einstein theory does not, and because of this one is able to provide a potential solution to the cosmological constant problem [2,3].

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Cosmological perturbations in conformal gravity. II.

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In this paper we continue a study of cosmological perturbations in the conformal gravity theory. In previous work we had obtained a restricted set of solutions to the cosmological fluctuation equations, solutions that were required to be both transverse and synchronous. Here we present the general solution. We show that in a conformal invariant gravitational theory fluctuations around any background that is conformal to flat (backgrounds that include the cosmologically interesting Robertson-Walker and de Sitter geometries) can be constructed from the (known) solutions to fluctuations around a flat background. For this construction to hold it is not necessary that the perturbative geometry associated with the fluctuations itself be conformal to flat. Using this construction we show that in a conformal Robertson-Walker cosmology early universe fluctuations grow as t^4. We present the scalar, vector, tensor decomposition of the fluctuations in the conformal theory, and we compare and contrast our work with the analogous treatment of fluctuations in the standard Einstein gravity theory.

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I. INTRODUCTION

In a recent paper [1] we presented the first steps in an analysis of cosmological fluctuations in the fourth-order derivative conformal gravity theory. Conformal gravity has been advanced by one of us as a candidate alternative to standard Einstein gravity, and reviews of its status at both the classical and the quantum levels may be found in [2–4], with the establishment of its unitarity and the positivity of its inner product at the quantum level being found in [5–8] and reviewed briefly in the Appendix G. Various other studies of conformal gravity and of higher derivative gravity theories in general can be found in [9–22]. In the study of [1] we found some specific perturbative solutions that are of cosmological interest, and in this paper we present the general and exact perturbative solutions to fluctuations around any background that is conformal to flat. Since both the Robertson-Walker and the de Sitter geometries are conformal to flat, our results are immediately of relevance to cosmology. As we show in both [1] and here, since Robertson-Walker and de Sitter background geometries are conformal to flat, the treatment of fluctuations around them is greatly facilitated by working in a gravitational theory that possesses conformal symmetry. In fact, by the judicious choice of gauge that we make in this paper (specifically a gauge condition that is itself conformally invariant), we are able to show that in the conformal theory fluctuations around any background that is conformal to flat can be constructed from fluctuations around a flat background, with this being the case even though the perturbative geometry associated with the fluctuations need not itself be conformal to flat.

As a possible candidate alternative to standard Einstein gravity, conformal gravity is attractive in that it is a pure metric theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of standard gravity while augmenting it with an additional symmetry, local conformal invariance, in which the action is left invariant under local conformal transformations on the metric of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Under such a symmetry a gravitational action that is to be a polynomial function of the Riemann tensor is uniquely prescribed, and with use of the Gauss-Bonnet theorem is given by (see e.g., [2])

$$I_{\mathrm{W}} = -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \equiv -2\alpha_g \int d^4x(-g)^{1/2} \left[R_{\mu\kappa} R^{\mu\kappa} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2 \right]. \quad (1)$$

Here α_g is a dimensionless gravitational coupling constant, and

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}) + \frac{1}{6} R^{\alpha}{}_{\alpha}(g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}) \quad (2)$$

is the conformal Weyl tensor, a tensor that vanishes in geometries that are conformal to flat, and that for any

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Radial Acceleration and Tully-Fisher Relations in Conformal Gravity

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Abstract. In 2016 McGaugh, Lelli and Schombert established a universal Radial Acceleration Relation for centripetal accelerations in spiral galaxies. Their work showed a strong correlation between observed centripetal accelerations and those predicted by luminous Newtonian matter alone. Through the use of the fitting function that they introduced, mass discrepancies in spiral galaxies can be constrained in a uniform manner that is completely determined by the baryons in the galaxies. Here we present a new empirical plot of the observed centripetal accelerations and the luminous Newtonian expectations, which more than doubles the number of observed data points considered by McGaugh et al. while retaining the Radial Acceleration Relation. If this relation is not to be due to dark matter, it would then have to be due to an alternate gravitational theory that departs from Newtonian gravity in some way. In this paper we show how the candidate alternate conformal gravity theory can provide a natural description of the Radial Acceleration Relation, without any need for dark matter or its free halo parameters. We discuss how the empirical Tully-Fisher relation follows as a consequence of conformal gravity.

1. Introduction

The rotation curves of spiral galaxies have been intensively studied for many years and serve as a key case study for the missing mass problem. Following the pioneering work of Freeman [1], of Roberts and Whitehurst [2], and of Rubin, Ford and Thonnard [3], there has been extensive study of the rotation curves of spiral galaxies, in particular in the region far beyond the optical disk. The cold dark matter (CDM) formalism, first developed for spiral galaxies by Navarro, Frenk and White (NFW) [4], explains mass discrepancies through the introduction of dark matter, to thus remain consistent with Newtonian gravity. Even with the ability of NFW to fit rotation curves, a lack of direct observational evidence for dark matter and the shrinking allowed parameter space still allowed for it invites alternate explanations of the missing mass problem. Some typical attempts are Modified Newtonian Dynamics (MOND) [5], Modified Gravity (MOG) [6], and Conformal Gravity (CG) [7]. Typical of these alternate theories is the ability not just to fit galactic rotation curves without dark matter, but to do so without the two

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Universal properties of galactic rotation curves and a first principles derivation of the Tully–Fisher relation

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ABSTRACT

In a recent paper McGaugh, Lelli, and Schombert showed that in an empirical plot of the observed centripetal accelerations in spiral galaxies against those predicted by the Newtonian gravity of the luminous matter in those galaxies the data points occupied a remarkably narrow band. While one could summarize the mean properties of the band by drawing a single mean curve through it, by fitting the band with the illustrative conformal gravity theory with fits that fill out the width of the band we show here that the width of the band is just as physically significant. We show that at very low luminous Newtonian accelerations the plot can become independent of the luminous Newtonian contribution altogether, but still be non-trivial due to the contribution of matter outside of the galaxies (viz. the rest of the visible universe). We present a new empirical plot of the difference between the observed centripetal accelerations and the luminous Newtonian expectations as a function of distance from the centers of galaxies, and show that at distances greater than 10 kpc the plot also occupies a remarkably narrow band, one even close to constant. Using the conformal gravity theory we provide a first principles derivation of the empirical Tully–Fisher relation.

1. Introduction

In a recent study McGaugh, Lelli, and Schombert (MLS) [1] presented an empirical plot of the observed centripetal accelerations ($g(OBS)$) of points in a wide class of spiral galaxies versus the luminous Newtonian expectations ($g(NEW)$) for those points. While the plot does not contain any information that is not already contained in plots of individual galactic rotation curves, the utility of the plot is that it allows one to include the data from every single galaxy in one and the same figure. The plot thus enables one to encapsulate a large amount of galactic rotation curve data in a single plot, doing so in a way that allows one to identify regularities in galactic rotation curve data that hold for all spiral galaxies. Inspection of the $g(OBS)$ versus $g(NEW)$ plot that we present in Fig. 1 immediately reveals three striking features. First, as noted by Milgrom in his development of the MOND theory [2], the departure from $g(NEW)$ first sets in when $g(OBS)$ drops below a universal acceleration scale of order 10^{-10} m s^{-2}. Second, when there are departures they occupy a remarkably small region in the plot. And third, as noted in [1], these departures would appear to be quite tightly correlated with the luminous Newtonian prediction. To quantify such a possible correlation, MLS made a one-parameter best mean fit to the plot in Fig. 1 with a fundamental acceleration parameter g_0, and found a good fit with the function $g(OBS) = g(MLS)$ where

$$g(MLS) = \frac{g(NEW)}{[1-\exp(-(g(NEW)/g_0)^{1/2})]}, \tag{1}$$

and extracted a value $g_0 = 1.20 \times 10^{-10}$ m s^{-2}. In this paper we shall evaluate the results of MLS and reach some alternate conclusions.

2. The data analysis

In trying to produce a $g(OBS)$ versus $g(NEW)$ plot there are two key variables, the distance to each galaxy (needed to fix distances R from galactic centers in $g(OBS) = v^2_{OBS}/R$), and the visible mass of each galaxy (needed for $g(NEW)$). While uncertainties

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RESEARCH ARTICLE

Three-dimensional and four-dimensional scalar, vector, tensor cosmological fluctuations and the cosmological decomposition theorem

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Abstract

In cosmological perturbation theory it is convenient to use the scalar, vector, tensor basis as defined according to how these components transform under three-dimensional rotations. In attempting to solve the fluctuation equations that are automatically written in terms of gauge-invariant combinations of these components, the equations are taken to break up into separate scalar, vector and tensor sectors, the decomposition theorem. Here, without needing to specify a gauge, we solve the fluctuation equations exactly for some standard cosmologies, to show that in general the various gauge-invariant combinations only separate at a higher-derivative level. To achieve separation at the level of the fluctuation equations themselves one has to assume boundary conditions for the higher-derivative equations. While spatially asymptotic boundary conditions suffice for fluctuations around a de Sitter background or a spatially flat Robertson–Walker background, for fluctuations around a spatially non-flat Robertson–Walker background one additionally has to require that the fluctuations be well-behaved at the origin. We show that in certain cases the gauge-invariant combinations themselves involve both scalars and vectors. For such cases there is no decomposition theorem for the individual scalar, vector and tensor components themselves as that would violate gauge invariance, but for the gauge-invariant combinations there still can be. Given the lack of manifest covariance (though not of covariance itself) in defining a basis with respect to three-dimensional rotations, we introduce an alternate scalar, vector, tensor basis whose components are defined according to how they transform under four-dimensional general coordinate transformations. With this basis the fluctuation equations greatly simplify, and while one can again break them up into separate gauge-invariant sectors at the higher-derivative level, in general we find that even with boundary conditions we do not obtain a decomposition theorem in which the fluctuations separate at the level of the fluctuation equations themselves.

Keywords Cosmological perturbation theory · Decomposition theorem · Four dimensional formalism

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Cosmological fluctuations on the light cone

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In studying temperature fluctuations in the cosmic microwave background Weinberg has noted that some ease of calculation and insight can be achieved by looking at the structure of the perturbed light cone on which the perturbed photons propagate. In his approach Weinberg worked in a specific gauge and specialized to fluctuations around the standard Robertson-Walker cosmological model with vanishing spatial three-curvature. In this paper we generalize this analysis by providing a gauge invariant treatment in which no choice of gauge is made, and by considering geometries with non-vanishing spatial three-curvature. By using the scalar, vector, tensor fluctuation basis we find that the relevant gauge invariant combinations that appear in the light cone temperature fluctuations have no explicit dependence on the spatial curvature even if the spatial curvature of the background geometry is nonvanishing. We find that a not previously considered, albeit not too consequential, temperature fluctuation at the observer has to be included in order to enforce gauge invariance. As well as working with comoving time we also work with conformal time in which a background metric of any given spatial three-curvature can be written as a time-dependent conformal factor (the comoving time expansion radius as written in conformal time) times a static Robertson-Walker geometry of the same spatial three-curvature. For temperature fluctuations on the light cone this conformal factor drops out identically. Thus the gauge invariant combinations that appear in the photon temperature fluctuations have no explicit dependence on either the conformal factor or the spatial three-curvature at all.

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I. INTRODUCTION

In analyzing the cosmological perturbations that can be measured in the cosmic microwave background (CMB) it is very convenient to use the scalar, vector, tensor (SVT) basis for the fluctuations as developed in [1,2]. In this basis the fluctuations are characterized according to how they transform under three-dimensional spatial rotations, and in this form the basis has been applied extensively in cosmological perturbation theory (see e.g., [3–8] and [9–13]). With the SVT expansion being based on quantities that transform as three-dimensional scalars, vectors and tensors, as such it is particularly well suited to Robertson-Walker geometries because such geometries have a spatial sector that is maximally three-symmetric. While not manifestly covariant, the scalar, vector, tensor expansion is covariant as it leads to equations that involve appropriate combinations of the scalars, vectors and tensors that are fully four-dimensionally (i.e., not just three-dimensionally) gauge invariant, this being all that one needs for covariance. Given that the fluctuation equations are gauge invariant one can of course work in any particular gauge that might be convenient. However, it is also informative to use a formalism that is manifestly fully gauge invariant throughout and that does not involve any specific choice of gauge at all [14]. Such an approach to cosmological perturbation theory has been followed in [15–17], and in this paper we apply this approach to light cone fluctuations in the CMB [18]. As we show, the relevant gauge invariant combinations that appear in the temperature fluctuations have no explicit dependence on the spatial curvature even if the spatial curvature of the background geometry is non-vanishing. This result meshes well with the observed CMB temperature fluctuations since in standard gravity they are found to favor vanishing spatial three-curvature. As well as work in comoving time we also work in conformal time where the background metric can be written as an overall time-dependent conformal factor times a static metric, and find that the photon temperature fluctuations are completely independent of the conformal factor. Thus, quite strikingly, light cone temperature fluctuations have no explicit dependence on either the conformal factor or the spatial three-curvature at all.

Our interest here is in fluctuations around cosmological backgrounds that are described by Robertson-Walker metrics of the form

$$\begin{aligned} ds^2 &= -g_{\mu\nu}dx^\mu dx^\nu \\ &= dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] \\ &= dt^2 - a^2(t)\tilde{\gamma}_{ij}dx^i dx^j, \end{aligned} \tag{1.1}$$

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Exact solution to perturbative conformal cosmology from recombination until the current era

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In a previous paper [P. D. Mannheim, Phys. Rev. D **102**, 123535 (2020)] we studied cosmological perturbation theory in the cosmology associated with the fourth-order derivative conformal gravity theory, and provided an exact solution to the theory in the recombination era. In this paper we present an exact solution that holds all the way from recombination until the current era.

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I. INTRODUCTION

A. Motivation

A primary interest of cosmological research has been the study of cosmological fluctuations around a homogeneous and isotropic cosmic microwave background (see, e.g., [1–5]). In this research the focus has been on the Einstein-gravity-based inflationary universe model [6], and this has led to a concordance model (see, e.g,. [7–9]) of a spatially flat universe composed predominantly of dark matter and dark energy. However, to date no dark matter candidates have actually been detected, and the required dark energy or cosmological constant 60 orders of magnitude fine tuning problem has yet to be resolved. Moreover, it is presumed that the classical gravity treatment of the concordance model that is made would not be destroyed by quantum gravitational radiative corrections even though they are uncontrollably infinite [10]. In response to these concerns, some candidate alternative proposals have been advanced in the literature. In this paper we consider one specific alternative, namely conformal gravity (see [11–16] and references therein). As a candidate gravitational theory conformal gravity has been shown capable of eliminating the need for galactic dark matter not only by providing fits to a wide class of galactic rotation curves without any need for dark matter but by doing so with universal galaxy-independent parameters [17–19]. In contrast, in dark matter fits one currently does have to introduce galaxy-dependent free parameters for each galactic dark matter halo. Also it was shown [11,14–16] that through its underlying local conformal symmetry [invariance under $g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)} g_{\mu\nu}(x)$ with a spacetime-dependent $\alpha(x)$] conformal gravity controls the cosmological constant without fine tuning. And with the conformal symmetry requiring that the gravitational sector coupling constant α_g be dimensionless, the conformal theory is power-counting renormalizable. With conformal gravity also being quantum-mechanically ghost free and unitary [20–23], conformal gravity provides a candidate quantum theory of gravity in four spacetime dimensions.

From the perspective of conformal gravity the dark matter, dark energy, and quantum gravity problems are not three separate problems at all. Rather, they all have a common cause, namely the extrapolation of Newton-Einstein gravity beyond its solar system origins. Consequently, they can all have a common solution, with conformal gravity endeavoring to provide such a solution through a different extrapolation of solar system wisdom. For cosmology the conformal theory has successfully been applied to the homogeneous and isotropic cosmological background. And it has been found [14–16,24] to yield a horizon-free background cosmology with no flatness problem, while providing a very good, non-fine-tuned, dark-matter-free fit to the accelerating universe supernovae data of [25,26]. However, conformal gravity still needs to be applied to the fluctuations around that background. Such a study is now in progress, with the initial development of the cosmological perturbation theory that is required having been presented in general in [27–31]. In this paper we take a further step by providing a new exact solution to the fourth-order derivative conformal gravity cosmological fluctuation equations that holds all the way from recombination until the current era. We had in fact already provided an exact solution that holds at recombination itself [30], and in this paper we build on that study. In regard to conformal gravity we note also that various other studies of conformal gravity and of higher derivative gravity theories in general can be found in [32–46].

In [30] we derived the conformal gravity cosmological fluctuation equations in all generality, for any background cosmological geometry and any set of background matter sources. While these equations hold for any background

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Inferring Space Object Orientation with Spectroscopy and Convolutional Networks

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ABSTRACT

Accurate inference of a space object's orientation is imperative for deriving its operational status and coordinating effective space traffic management at large. To formulate the framework necessary for solving the problem of orientation inference, we analyze several standard mathematical representations of rotation with an emphasis on continuity, uniqueness, and deep learning efficacy. On this basis, we are naturally led to the implementation of a lesser known but well-behaved 6D representation of rotation. For the input of our inference models, we employ a distance-invariant observational technique that has long been used to probe the furthest reaches of the universe at the smallest scales – spectroscopy. Facilitated by deep convolutional neural networks (CNN's), we investigate the viability of using simulated raw, long slit spectroscopic images to infer the orientation of space objects in the nonresolved regime of large orbital radii. We present methods and results of training CNN's on spectral images of several space objects with an aim to i) standardize the measures used in rotation analysis, ii) establish an upper bound on spectral-based performance, and iii) provide a simple-scenario baseline for the extension of future work in the application of spectroscopy to space domain awareness.

1. INTRODUCTION

One of the fundamental goals in the field of space domain awareness (SDA) is to provide an increasingly complete specification of the physical state and trajectory associated with a target space object orbiting Earth. While the set of six Keplerian elements (commonly encoded as 'two-line elements') form a sufficient basis to describe the unperturbed orbit of the center of mass, additional degrees of freedom remain concerning the motion of a body about its center of mass. To continue advancing towards a more complete description of the physical state, one must also specify the rotational kinematics of the body, namely its orientation and angular velocity relative to a given reference frame. Increasing the complexity further, if the body itself is not rigid, yet even more degrees of freedom must be determined to account for arbitrary articulations of mass extended by the space object.

In this work, we focus on a specific subset of the aforementioned degrees freedom – rotational orientation. We approach orientation inference as a computer vision task implemented by deep convolutional networks. The description of rotation itself and its generalization to both *n* dimensions and complex fields has a strikingly rich foundation in both mathematics and physics, serving as an integral component in the study of group theory (Lie groups and Lie algebras) and the description of fundamental symmetries that form the basis of modern theoretical physics. In light of this vast groundwork, in Sect. 3 we pay due diligence to investigate numerous mathematical representations of rotation and assess their utility within deep learning applications. The selection of the appropriate representation of rotation will serve to define the framework by which we approach the task of learning orientation and will provide a basis for measuring model performance.

The physical scenario we target broadly concerns ground based observation of resident space objects located at orbital radii too large to form an optically 'resolved' image. For the purposes of this work, our observations are modeled after the 3.67 meter Advanced Electro Optical System Telescope (AEOS) stationed at the summit of Mt. Haleakala, Maui. To provide an intuition of how optical resolvability varies with altitude, Fig. 1 (right) shows two images of the Hubble spacecraft as simulated through a high-resolution adaptive optics sensor on AEOS. In Fig. 1 (top right) we

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Influence of Force-Constant Changes on the Lattice Dynamics of Cubic Crystals with Point Defects*

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The crystal-point impurity problem is examined for the case of changes in nearest-neighbor forces around the defect site. Using the recently developed group-theoretical techniques, expressions are found for the frequencies of the perturbed modes due to impurities in face-centered and body-centered cubic crystals, assuming only central forces. By solving coupled equations for double-time Green's functions, expressions are also obtained for the defect mean-square displacement and its amplitude of vibration in each perturbed mode. Applications are then made to recent optical and Mössbauer measurements.

I. INTRODUCTION

IT is well known[1] that in the presence of defects great changes in the dynamical properties of a crystal may be obtained. The subject has been treated by many authors and solutions have been given for the determination of localized modes produced by a point mass defect. The method is to use the localized nature of the perturbation to the pure crystal and solve using the lattice Green's functions. This is possible because we have only added a source term to the pure-crystal equations of motion. By the same method it is also possible to calculate the amplitude of vibration of the defect in each of the normal modes of the perturbed crystal. This is very useful because in optical measurements, neutron scattering, and Mössbauer experiments, it is the defect motion which determines the process.

Though the isotopic impurity has been very popular for illustrative purposes, not too much attention has been given to considerations of the influence of changes in force constants in the vicinity of the defect. This is somewhat unreasonable since in actual experiments forces are usually changed as well. There have been some calculations performed for changes of force in a nearest-neighbor simple cubic lattice.[2] This however is an unphysical situation in which the polarizations of the phonons are ignored and the lattice becomes one-dimensional. An important development was the introduction of the matrix partitioning techniques,[3] which admit of changes in the force constants and also take into account the polarizations of the phonons. In the last two years group-theoretical techniques have become generally available[4] so that it is possible to solve for crystals with a more complicated lattice structure. The use of symmetry adapted coordinates is now thought to be the most convenient approach to the problem, and applications are now appearing regularly in the literature.[5–9]

In this paper we shall examine the defect problem for changes in nearest-neighbor forces due to impurities substitutionally inserted into face-centered and body-centered cubic lattices. We obtain conditions for the determination of the position of localized and resonance modes, and the defect amplitude of vibration $|\chi^2(0,\omega)|$ as a function of the normal mode frequency ω [Eqs. (32) and (55), respectively].

For the case of central forces only, a simple formula (32) is given which only requires knowledge of the pure-crystal density of states for solution. It contains two free parameters, the mass change M/M', and the change in the force constant at the defect site $A_{xx}(0,0)/A_{xx}'(0,0)$.

We discuss first, in Sec. II, properties of the pure crystal which are necessary for solving the impurity problem. We then examine in Sec. III the cluster of atoms around the defect where changes take place. For instance, for a face-centered cubic lattice we have 13 atoms in the cluster and hence 39 degrees of freedom. Thus to get the frequencies we have to diagonalize a 39 dimensional matrix. This is treated in Sec. IV. We make a block diagonalization of this matrix using the lattice site symmetry and isolate the F_{1u} mode in which the defect moves. We then have a four-dimensional matrix containing a large number of lattice Green's functions. We next derive relations between the Green's functions to reduce their number. The perturbed frequencies are derived in Secs. IV and V for the fcc and bcc crystals, respectively. To obtain the defect mean-square displacement $\langle u^2(0)\rangle$, we introduce double-time

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Influence of Force-Constant Changes and Localized Modes on the V:Fe^{57} Mössbauer System*

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An experimental and theoretical study is made of the Mössbauer effect of Fe^{57} in a vanadium matrix. We have measured the recoil-free fraction and second-order Doppler shift over the temperature range 100 to 700°K. The experimental results are interpreted using recently derived expressions for the mean-square velocity and displacement of the defect which allow for changes in the force constants as well as the mass at the impurity site. A good fit to the data is obtained for an increase in force constant of approximately 2.5. The data are also analyzed in the high-temperature limit to confirm this increase. Evidence for the presence of localized modes is found from the velocity shift, and the positions of the modes are determined.

1. INTRODUCTION

THE probability for the Mössbauer effect, its temperature dependence, and the temperature dependence of the second-order Doppler shift of the Mössbauer line are directly related to the dynamics of the Mössbauer-active nucleus.[1] When this nucleus is an impurity placed in a crystal, the lattice dynamics of the composite system influences the determination of the recoil-free fraction and the second-order Doppler shift. However, since an impurity has been introduced into the crystal, the dynamics is no longer that of the pure crystal. In particular there is a change in mass at the defect site and changes in force constants in the vicinity of this site. This perturbation can affect the mean-square displacement $\langle x^2\rangle$ and the mean-square velocity $\langle v^2\rangle$ of the impurity atom.

The evaluation of these quantities has been achieved for the case of an isotopic impurity.[1] Ignoring the change in force, the temperature dependence of the recoil-free fraction of Sn^{119} in V has been satisfactorily fitted.[2] For such calculations the essential input data is the density of states of the pure crystal, which is usually known experimentally. The influence of force-constant changes has not been taken into account quantitatively, except in the somewhat unphysical model of a simple cubic lattice with nearest-neighbor interactions only.[3] Recently[4] closed expressions for $\langle x^2\rangle$ and $\langle v^2\rangle$ have been derived for the central force body- and face-centered cubic crystal models in which there are changes in the nearest-neighbor forces around the defect site. Again the only required input data is the pure-crystal phonon spectrum.

Using these expressions it is then possible to fit the experimental data treating the force-constant ratio λ'/λ as a free parameter to be determined. As well as detailed fitting of the data at all temperatures, it is possible to obtain information from macroscopic fitting at the high-temperature limit. This is valuable since it serves as an indication to the force changes, and is also model-independent.

We have studied the system of Fe^{57} in V both experimentally and theoretically. In Sec. 2 we describe the theoretical situation at the microscopic and macroscopic levels. In Sec. 3 we describe the experimental determination of the recoil-free fraction and second-order Doppler shift and analyze the results in Sec. 4, where we find definite evidence for a big force-constant increase, and for the presence of localized modes. In Sec. 5 we discuss the sensitivity of the calculation to the various input parameters.

2. THEORETICAL ANALYSIS

The probability (f) of recoil-free γ-ray emission is given by

$$f=|\langle i|\exp(i\boldsymbol{\kappa}\cdot\mathbf{r})|i\rangle|^2, \tag{1}$$

where $|i\rangle$ represents the initial and hence final state of the lattice. Here $\mathbf{r}$ is the position of the emitting nucleus and $\boldsymbol{\kappa}$ is the wave vector of the γ ray. Lipkin[5] has shown that under certain conditions, which we discuss later, we may write

$$f=\exp(-\kappa^2\langle x^2\rangle). \tag{2}$$

$\langle x^2\rangle$ is the mean-square displacement of the nucleus

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Local vibrational densities of states of dilute Fe atoms in Al and Cu metals

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The local vibrational densities of states of highly dilute Fe (0.017 at. %) in Al metal and Fe (0.1 at. %) in Cu metal were measured by using nuclear resonant inelastic scattering of synchrotron radiation. The characteristic vibrational modes of Fe in Al were found to be modes of resonance with the host Al phonon spectrum. In the measured spectrum of Fe in Cu, in addition to resonance vibrational modes, just above the phonon cutoff energy of Cu metal a peak interpreted as being a localized mode predicted theoretically by the Green's-function method was found.

I. INTRODUCTION

It is widely known that the presence of impurity atoms in a material influences its electronic and/or thermal properties. In the study of these properties, Mössbauer spectroscopy is an effective tool because it allows the characterization of the local surroundings of the impurity atom. In addition to information on the local electronic state, information on the intrinsic dynamic nature of the impurity can be obtained from recoilless fraction measurements.[1] Theoretically, the dynamics of an impurity atom has been studied using the Green's-function technique,[1,2] and theories adopting this technique have been applied to derive the modified vibrational density of states for an impurity atom from the unperturbed phonon density of states. In general, these studies indicate the existence of resonance modes whose frequencies lie in the range of the normal modes of the unperturbed host crystal. Furthermore, in cases where the mass of an impurity atom is sufficiently light and/or the binding of an impurity atom to the host crystal is sufficiently strong, they show that there may be a localized mode, whose frequency is greater than the maximum frequency of the unperturbed crystal. In the case of dilute substitutional Fe atoms in Al metal, the vibrational density of states was expected to be a resonance type from Mössbauer recoilless fraction measurements.[3,4] On the other hand, in the case of dilute substitutional Fe atoms in Cu metal, the existence of a localized mode was predicted,[4,5] but it has not yet been observed.

Though Mössbauer spectroscopy is effective, it is difficult to observe directly the vibrational density of states of an impurity atom, which is exceedingly important in the study of the thermodynamic properties of a material. In cases involving a highly dilute impurity, observation of the vibrational density of states is difficult even by other methods of measurement such as neutron inelastic scattering, with some exceptions. Direct observation is possible, however, by using nuclear resonant inelastic scattering of synchrotron radiation, the applicability and effectiveness of which was shown in Ref. 6. One of the most important features of this method is the possibility of measurement of a specific element and therefore the observation of the intrinsic motion of doped or intercalated atoms is possible.

We have measured the local vibrational spectra of substitutional impurity Fe atoms in Al and Cu metals by using nuclear resonant inelastic scattering and we obtained the impurity vibrational densities of states from the measured spectra. In this paper, we report the results and discuss them by means of the Green's-function method developed by Mannheim and co-workers.[7]

II. EXPERIMENT

To prepare an Al-0.017 at. % Fe ($Fe_{0.00017}Al_{0.99983}$) foil specimen, an alloy ingot was made by arc melting a 99.9999% Al foil and metallic ^{57}Fe powder (95.6% enriched) under an argon atmosphere; it was homogenized for 24 h in

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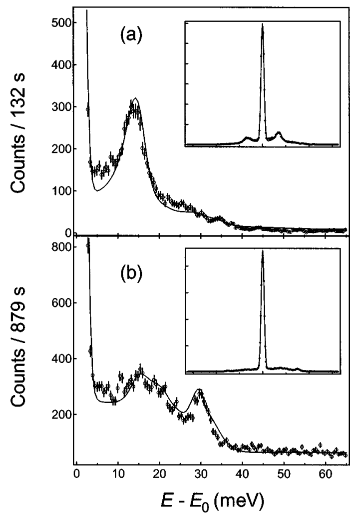

FIG. 2. Nuclear resonant inelastic scattering spectra of synchrotron radiation by ^{57}Fe in (a) Al-0.017 at. % Fe and (b) Cu-0.1 at. % Fe. Solid curves were obtained through least-squares fit of the data to the nuclear resonant excitation cross section σ_r (Refs. 6 and 16) which is calculated on the basis of Mannheim's impurity theory (Refs. 1 and 7) and convoluted with the resolution function of the monochromator; the ratio of the effective host-host force constant to the impurity-host constant was obtained to be 0.94 for (a) Al-0.017 at. % Fe and 0.79 for (b) Cu-0.1 at. % Fe. The incident photon energy and the energy of the first excited state of ^{57}Fe are denoted as E and E_0.

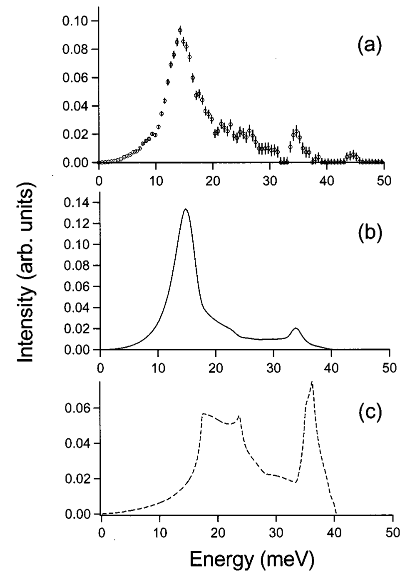

FIG. 3. (a) Vibrational density of states of ^{57}Fe in Al-0.017 at. % Fe obtained from the corresponding measured nuclear resonant inelastic scattering spectrum [Fig. 2(a)]. (b) Response function of the impurity Fe atom to the phonon density of states of Al metal (Ref. 13) calculated on the basis of Mannheim's impurity theory (Refs. 1 and 7); in this calculation, the ratio of the effective host-host force constant to the impurity-host constant was taken to be 0.94. (c) Unperturbed phonon density of states of Al (Ref. 13).

developed by Mannheim and co-workers and Grow et al.,[1,7] which applies to an isolated impurity with central forces limited to the first nearest neighbors. This theory has an analytical form and contains only one free parameter A/A' which is the ratio of the host-host force constant to the impurity-host constant. This parameter and the known host-impurity mass ratio M/M' determine the coupling of the impurity to the lattice vibrations of the host metal. The response of the impurity to the phonon density of states of the host $G(\omega)$ is expressed as the modified vibrational density of states $G'(\omega)$ as follows:

$$G'(\omega)=(M/M')G(\omega)\{[1+\rho(\omega)S(\omega)]^2 +[(\pi/2)\omega G(\omega)\rho(\omega)]^2\}^{-1}+\delta(\omega-\omega_L)(M/M') \times\{\rho^2(\omega)T(\omega)+(M/M')-[1+\rho(\omega)]^2\}^{-1}, \quad (1)$$

where

$$\rho(\omega)=(M/M')-1+\omega^2[1-(A/A')]/\mu(+2), \quad (2)$$

$$S(\omega)=\mathcal{P}\int_0^\infty \omega'^2(\omega'^2-\omega^2)^{-1}G(\omega')d\omega', \quad (3)$$

$$T(\omega)=\omega^4\int_0^\infty(\omega'^2-\omega^2)^{-2}G(\omega')d\omega', \quad (4)$$

$$\mu(n)=\int_0^\infty \omega^n G(\omega)d\omega, \quad (5)$$

and ω_L denoting a localized mode frequency exists only if it satisfies that $1+\rho(\omega_L)S(\omega_L)=0$ and $\omega_L>\omega_{\max}$; $\delta(\omega-\omega_L)$ is the Dirac δ function and $\mathcal{P}$ stands for principal value. We made the least-squares fit of the data to the nuclear resonant excitation cross section σ_r,[6,16] which is calculated from $G'(\omega)$ and convoluted with the resolution function of the monochromator. For the calculation of $G'(\omega)$ of the dilute substitutional Fe atom in Al and Cu, we used the unperturbed phonon densities of states $G(\omega)$ of Al (Ref. 13) and Cu (Ref. 14), respectively; they were calculated from the results of neutron inelastic scattering experiments. It should be noted that the force-constant ratio A/A' is the effective ratio, since the unperturbed phonon densities of states are obtained based on the results of experiments on neutron inelastic scattering measured at room temperature, and these results may contain anharmonic effects.[17] In the theory, a localized mode has an infinite lifetime and is expressed as a


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Measuring velocity of sound with nuclear resonant inelastic x-ray scattering

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Nuclear resonant inelastic x-ray scattering is used to measure the projected partial phonon density of states of materials. A relationship is derived between the low-energy part of this frequency distribution function and the sound velocity of materials. Our derivation is valid for harmonic solids with Debye-like low-frequency dynamics. This method of sound velocity determination is applied to elemental, composite, and impurity samples which are representative of a wide variety of both crystalline and noncrystalline materials. Advantages and limitations of this method are elucidated.

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Mechanical properties form an important part of our understanding of condensed matter. In many areas of science, measurements of sound velocity are used to study materials of both natural occurence and artificial fabrications. For example, in the field of geophysics, the sound velocity is the most direct information we have about the Earth's interior. The standard approach to learn about the composition and structure of the Earth's interior entails measurements of sound velocities of candidate compounds. The results are then compared to seismological data to exclude or confirm a particular compound. In the following, we will describe the use of nuclear resonant inelastic x-ray scattering (NRIXS) to measure the velocity of sound.

The NRIXS method was introduced to probe the lattice dynamics of materials by employing low-energy nuclear resonances.[1,2] In NRIXS experiments, only signals from nuclear resonance absorption are monitored, and for this reason the extracted quantity is specific to the resonant isotope. This technique provides the phonon excitation spectrum as seen by the probe nuclei,[3–5] and in most cases one can extract the partial vibrational frequency distribution, a function often referred to as the partial phonon density of states (PDOS). The NRIXS method has been applied to various materials, e.g., thin films and multilayers,[6–8] nanoparticles,[9,10] crystals with impurities,[11] organic molecules,[12–15] proteins,[16,17] samples under high pressures,[18] and samples of geophysical interests.[19,20] Most of these samples are compounds, and, while the obtained PDOS gives only part of the lattice dynamics, the low-energy portion of the PDOS provides the Debye sound velocity of the whole sample. We will now show that, due to universal features of acoustic modes of harmonic solids, the low-energy portion of the PDOS is related to the Debye sound velocity in a simple way.

The normalized phonon density of states is defined by

$$\nu(E)=\frac{1}{3N}\sum_{l}^{3N}\delta(E-E_l), \tag{1}$$

where the energy eigenstates of lattice vibrations E_l are labeled by quantum number l, and N is the total number of atoms in the solid. In the harmonic lattice approximation, a PDOS, which is more relevant to NRIXS experiments, is given by[3]

$$\mathcal{D}(E,\hat{\mathbf{k}})=\frac{1}{\tilde{N}}\sum_{\nu=1}^{\tilde{N}}\frac{1}{N}\sum_{l=1}^{3N}|\hat{\mathbf{k}}\cdot\mathbf{e}_l^{\nu}|^2\delta(E-E_l), \tag{2}$$

where ν enumerates resonant nuclei, $\tilde{N}$ is the total number of resonant nuclei, $\hat{\mathbf{k}}$ is a unit vector in the incident photon direction, and $\mathbf{e}_l^{\nu}$ are phonon polarization vectors. Equation (2) shows that the vibrational polarizations are projected onto the incident photon direction and in particular the vibrational modes with polarization perpendicular to the direction of the incident photon do not contribute. In the case of a single crystal, the measured vibrational properties become dependent on the incident photon direction and were called "projected,"[21,22] whereas in cases of polycrystalline or isotropic samples, the measured spectrum is an average over all directions. For a crystal in which resonant nuclei occupy only equivalent lattice sites, the quantity that can be extracted from NRIXS experiments is exactly described by Eq. (2). When the resonant nuclei occupy different sites, what can be extracted is an approximation of Eq. (2). The approximation is based on an average of phonon spectra for these different lattice sites. The closure conditions of the phonon polarization vectors guarantee the normalization of $\mathcal{D}(E,\hat{\mathbf{k}})$, i.e., its integration over all phonon energies is 1. The orthonormality and closure conditions are given by

$$\frac{1}{N}\sum_{\mu=1}^{N}\sum_{\alpha=1}^{3}(e_l^{\mu\alpha})^*e_{l'}^{\mu\alpha}=\delta_{ll'}, \tag{3}$$

$$\frac{1}{N}\sum_{l=1}^{3N}(e_l^{\mu\alpha})^*e_l^{\nu\beta}=\delta_{\mu\nu}\delta_{\alpha\beta}, \tag{4}$$

where α and β denote the spatial components. These conditions hold for any harmonic solid, and the polarization vec-

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Bounds on localized modes in the crystal impurity problem

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Using general properties of the crystal site representation normal mode matrix, we provide some very simple bounds on localized modes in simple, body-centered, and face-centered cubic crystals with substitutional point defects. We derive a trace condition constraint on the net change in crystal eigenfrequencies caused by the introduction of a defect, with the condition being a completely general one which holds for any combination of central and noncentral crystal force constants and for all-neighbor interactions. Using this condition we show that the sufficient condition for producing localized modes in an arbitrary cubic crystal by a mass change at the defect site is that the defect mass be less than one half of that of the host atom mass which it replaces, and that the sufficient condition for producing localized modes in an arbitrary cubic crystal by force-constant changes alone is that the defect site self-force constant be greater than twice that of the pure crystal self-force constant of the host atom which it replaces.

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I. INTRODUCTION

The substitutional insertion of a point defect impurity into an otherwise perfect host crystal will typically modify the spectrum of the $3N$ normal modes of the crystal,[1] leading in certain circumstances (such as the insertion of impurities which are lighter in mass than the host atoms which they replace or which are more strongly coupled to the host crystal atoms than the ones they replace) to the generation of modes with frequencies which lie beyond the band maximum $\omega_{\max}$ of the crystal. Such modes will not be plane waves which propagate throughout the crystal, but will instead fall off exponentially fast away from the defect site and thus be localized to it. Moreover, with the rest of the crystal atoms not participating appreciably in such localized modes, the intensity of the defect in such modes will be N times larger than the intensity it would otherwise have had in a crystal plane wave mode, to thus give the localized mode enough intensity to render it observable. While such modes could be of relevance for phenomena such as the Mössbauer effect associated with the insertion of Mössbauer active defects into host crystals, historical recoil-free fraction Mössbauer studies only involved an averaging over all the lattice modes of the system, to thereby only allow one to infer the possible presence of localized modes indirectly. However, with the advent of dedicated synchrotron rings it became possible to monitor Mössbauer active systems mode by mode directly; and via nuclear resonant inelastic x-ray scattering studies, modes lying beyond the host crystal phonon band maximum have now explicitly been seen in the $^{57}Fe/Cu$ and the $^{57}Fe/NiAl$ systems.[2,3] Consequently, knowing whether the insertion of a defect into pure crystal hosts might generate localized modes can be of great value for such studies.

In this paper we use a very straightforward trace technique to enable us to identify some very general conditions under which such localized modes can be produced. We shall restrict our study to the most straightforward case of single point defects which are substitutionally inserted in the three primitive simple, body-centered, and face-centered cubic crystals (these specific cases being amongst the most commonly experimentally studied ones), though with our approach being quite generic, it could in principle be adapted to encompass other crystal structures as well if desired. We shall provide results for various combinations of mass and force-constant changes, some which are specific to nearest-neighbor force constants and some of which are general to all-neighbor force constants. In Sec. II we derive the trace condition on which all of our results are based, with the provided relation [Eq. (15) below] being an exact, all-neighbor relation which permits an arbitrary mass change (M replaced by M') at the defect site and arbitrary force-constant changes between the defect atom and any other atom in the entire crystal, despite which the resulting relation only involves the self-force constant $A_{xx}(0,0)$ at the defect site and the change $A'_{xx}(0,0)$ in it. In Sec. III we apply our trace condition to the case of an isotopic substitution where the only change is that in the mass at the defect site, to show that while having a lighter impurity ($M'<M$) is necessary for the generation of a localized mode, it is the condition $M'<M/2$ which is the sufficient one. In Secs. IV and V we apply our trace condition to some straightforward nearest-neighbor force-constant change cases for which there are extremely simple exact analytic solutions to the impurity problem (some typical examples of which being crystals with force constants which are central or which are isotropic), to find that for all of them the sufficient condition for localized mode production by force-constant changes alone is given as the requirement that $A'_{xx}(0,0)/A_{xx}(0,0)$ be greater than $3/2$. Finally in Sec. V we examine more complicated force-constant change cases and go beyond nearest-neighbor force constants, and while these situations do not admit of as straightforward a treatment as the cases considered in Sec.

Classical underpinnings of gravitationally induced quantum interference

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We show that the gravitational modification of the phase of a neutron beam [the Colella-Overhauser-Werner (COW) experiment] has a classical origin, being due to the time delay that classical particles experience in traversing a background gravitational field. Similarly, we show that classical light waves also undergo a phase shift in traversing a gravitational field. We show that the COW experiment respects the equivalence principle even in the presence of quantum mechanics. [S1050-2947(98)03802-5]

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In a landmark series of experiments [1,2] Colella, Overhauser, and Werner and subsequent workers (see, e.g., [3–5] for overviews) detected the modification of the phase of a neutron beam as it traverses the Earth's gravitational field, to thus realize an experiment that involved both quantum mechanics and gravity. A typical generic experimental setup is shown in the schematic Fig. 1, in which a neutron beam from a reactor is Bragg split at point A into a horizontal beam AB and a vertical beam AC (we take the Bragg angle to be 45° for illustrative simplicity in the following), with the subsequent scatterings at B and C then producing beams that interfere at D, after which they are then detected. If the neutrons arrive at A with velocity v_0 ($v_0 \sim 2.8\times10^5$ cm sec^{-1} is typical [4]) and $ABCD$ is a square of side H (~ 4.8 cm), then the phase difference $\phi_{COW}=\phi_{ACD}-\phi_{ABD}$ is given by $-mgH^2/\hbar v_0$ to lowest order in the acceleration g due to gravity [1] and is actually observable despite the weakness of gravity, since even though $\int \bar{p}\, d\bar{r}$ differs only by the very small amount $m(v_{CD}-v_{AB})H=-mgH^2/v_0$ between the CD and AB paths, nonetheless this quantity is not small compared to Planck's constant, to thus give an observable fringe shift (~ 56.5 rad [4]) even for H as small as a few centimeters.

The detected Colella-Overhauser-Werner (COW) phase is extremely intriguing for two reasons. First, it shows that it is possible to distinguish between different paths that have common end points, with the explicit global ordering in which the horizontal and vertical sections are traversed leading to observable consequences. Second, it yields an answer that explicitly depends on the mass of the neutron even while the classical neutron trajectories (viz., the ones explicitly followed by the centers of the wave packets of the quantum-mechanical neutron beam) of course do not. The COW result thus invites consideration of whether the detected ordering is possibly a topological effect typical of quantum mechanics and of whether quantum mechanics actually respects the equivalence principle. As we shall see, the ordering effect is in fact already present in the motion of classical particles in gravitational fields and even in the propagation of classical waves in the same background, with this latter feature enabling us to establish below that the mass dependence of the neutron beam COW phase is purely kinematic with the equivalence principle then not being affected.

To address these issues specifically we have found it convenient to carefully follow the neutron as it traverses the interferometer, to find that the two beams do not in fact arrive at the same point D or even at the same time, with this spatial offset and time delay not only producing the interference effect, but also being present in the underlying classical theory. Quantum mechanics thus does not cause the time delay; rather it only serves to make it observable. Since gravity is a relativistic theory we shall need to introduce curvature (which we do below), but we have found it more instructive to consider the nonrelativistic limit first. Since we can treat the neutron beams as rays, their motions round the $ABCD$ loop can be treated purely classically between the various scatterings. Moreover, the various scatterings themselves at A, B, C, and D introduce no additional phases, are energy conserving, and give angles of reflection equal to the angles of incidence [6]. Thus the entire motion of the neutron is the same as that of a spinless macroscopic particle that undergoes classical mirror reflections.

A nonrelativistic classical neutron that goes up vertically from A arrives at C with a velocity $(0,v_0-gH/v_0)$. The neutron AC travel time is $t(AC)=(H+\delta)/v_0$ (where $\delta=gH^2/2v_0^2$) and the standard nonrelativistic classical action $S_{CL}=\int(\bar{p}\,d\bar{r}-E_0dt)$ $(E_0=mv_0^2/2)$ undergoes a change $S(AC)=mv_0(H-\delta)-E_0t(AC)$. On scattering at C the neutron is then reflected so that it starts off toward D with a velocity $(v_0-gH/v_0,0)$. On its flight it dips slightly to arrive at the next scattering surface at the point D_1 with coordinates $(H-\delta,H-\delta)$, so that there is a change in the end point of the motion that is first order in g and thus relevant to our

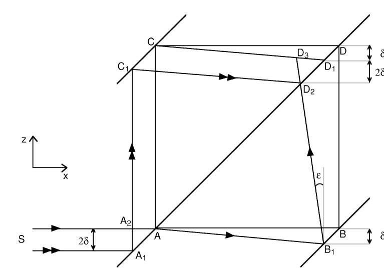

FIG. 1. COW wave paths.

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discussion. At D_1 the neutron has a velocity $(v_0 - gH/v_0, -gH/v_0)$, with the CD_1 segment taking a time $t(CD_1) = (H+\delta)/v_0$ and contributing an amount $S(CD_1) = mv_0(H-3\delta) - E_0 t(CD_1)$ to S_{CL}. A classical neutron that starts horizontally from A arrives not at B but at the point B_1 with coordinates $(H-\delta, -\delta)$ and with a velocity $(v_0, -gH/v_0)$. The AB_1 segment takes a time $t(AB_1) = (H-\delta)/v_0$ and the action changes by $S(AB_1) = mv_0(H-\delta) - E_0 t(AB_1)$. After scattering at B_1 the neutron sets off toward D with velocity $(-gH/v_0, v_0)$ and arrives not at D or D_1 but rather at the point D_2 with coordinates $(H-3\delta, H-3\delta)$ and reaches there with velocity $(-gH/v_0, v_0 - gH/v_0)$. The B_1D_2 segment takes a time $t(B_1D_2) = (H-\delta)/v_0$ and the action changes by $S(B_1D_2) = mv_0(H-3\delta) - E_0 t(B_1D_2)$.

As regards the neutron's path around the loop, we see from Fig. 1 that the small vertical dip δ during each of the two horizontal legs causes both of these neutron paths to be an amount δ shorter in the horizontal than they would have been in the absence of gravity, to thus provide a first order in g modification to $\int \bar{p}\ d\bar{r}$ in each of these legs, even while these same vertical dips themselves only contribute to the action in second order. However, for the two horizontal sections, each leg is shortened by the same amount in the horizontal, so that the difference in $\int \bar{p}\ d\bar{r}$ between the CD_1 and AB_1 legs still takes the value $-mgH^2/v_0$ quoted earlier. As regards the two vertical legs, we note that even though the AC leg is completely in the vertical, since the neutron beam starts the B_1D_2 leg with a small horizontal velocity, during this leg the neutron beam changes its horizontal coordinate by an amount 2δ, thereby causing it to reach D_2 after having also traveled a distance 2δ less in the vertical than it would travel in the AC leg. Consequently, there is both a spatial offset $(2\delta, 2\delta)$ between D_1 and D_2, and a time delay $t(ACD_1) - t(AB_1D_2) = 4\delta/v_0$ between the arrival of the two beams, with $\int \bar{p}\ d\bar{r}$ thus not taking the same value in each of the two vertical legs. However, our calculation shows that all these modifications actually compensate in the overall loop with there being no difference in $\int \bar{p}\ d\bar{r}$ between the ACD_1 and AB_1D_2 paths. However, even though there is still a net change in the action $S(ACD_1) - S(AB_1D_2) = -mgH^2/v_0$ because of the net time delay, we cannot identify this quantity with the COW phase $\hbar\Delta\phi_{COW}$ since the beams have not interfered due to the spatial offset between D_1 and D_2.

Before discussing the issue of this spatial offset, it is instructive to ask where the classical neutron paths would have met had there been no third crystal at D to get in the way. Explicit calculation shows that the paths would in fact have met at the asymmetric point D_3 with coordinates $(H-3\delta, H-\delta)$ with the CD_3 and B_1D_3 segments taking times $t(CD_3) = (H-\delta)/v_0$ and $t(B_1D_3) = (H+\delta)/v_0$, respectively, while yielding action changes $S(CD_3) = mv_0(H-5\delta) - E_0 t(CD_3)$ and $S(B_1D_3) = mv_0(H-\delta) - E_0 t(B_1D_3)$. The neutron paths would thus meet at D_3 without any time delay and with $S(ACD_3) - S(AB_1D_3) = -2mgH^2/v_0$. We thus see that for purely classical particles reflecting off mirrors at B_1 and C the quantity $\int \bar{p}\ d\bar{r}$ evaluates differently for the two paths ACD_3 and AB_1D_3.

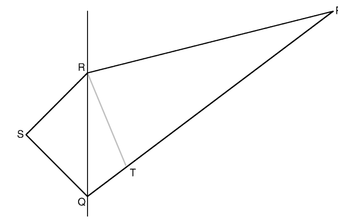

FIG. 2. Double-slit wave paths.

This is thus a global, path-dependent effect in purely classical mechanics in a background classical gravitational field that is completely independent of quantum mechanics [7]. However, since the classical action is not observable in classical mechanics, it is only in the presence of quantum mechanics that phase differences become observable. (In classical mechanics what is observable is that the neutron paths meet at D_3 rather than on the AD axis.)

Returning now to the COW experiment itself, in order to understand the implications of the time and spatial offsets between D_1 and D_2, it is instructive to consider the Young double-slit experiment with purely classical light. As shown in Fig. 2, light from a source S goes through slits Q and R to form an interference pattern at points such as P, with the distance $\Delta x = QT$ representing the difference in path length between the two beams. Given this path difference, the phase difference between the two beams is usually identified as $k\Delta x$, from which an interference pattern is then readily calculated. However, because of this path difference, the SQP ray takes the extra time $\Delta t = \Delta x/c$ to get to P, to thus give a net change in the phase of the SQP beam of $k\Delta x - \omega\Delta t$, which actually vanishes for light rays. The relative phase of the two light rays in the double-slit experiment thus does not change at all as the two beams traverse the interferometer. However, because of the time delay, the SRP beam actually interferes with an SQP beam that had left the source a time Δt earlier. Thus, if the source is coherent over these time scales, the SQP beam carries an additional $+\omega\Delta t$ phase from the very outset. This phase then cancels the $-\omega\Delta t$ phase it acquires during the propagation to P (a cancellation that clearly also occurs for quantum-mechanical matter waves moving with velocities less than the velocity of light), leaving just $k\Delta x$ as the final observable phase difference, a quantity that is nonzero only if there is in fact a time delay. We thus see that the double-slit device itself actually produces no phase change for light. Rather, the choice of point P on the screen is a choice that selects which time delays at the source are relevant at each P, with the interference pattern thus not only involving a time delay at the source, but in fact even requiring one.

With this in mind, we now see that we also need to monitor the time delay of the neutron in the COW experiment. However, since the total energy of the neutron does not change as it goes through the interferometer, the time-delay contribution will still drop out of the final phase-shift expression (explicitly but not implicitly). However, for the COW experiment we noted above that as well as a time delay bethe general R_ξ formalism, see Ref. 5, where the second-order scalar-particle self-energy with an arbitrary value of ξ is also evaluated.

[8]This problem has been investigated, with the use of conventional formalism for the vector field, by T. Appelquist and H. Quinn, Phys. Lett. 39B, 229 (1972).

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Infrared bootstrap for the electron mass in finite quantum electrodynamics

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This paper is concerned with the infrared structure of Johnson-Baker-Willey finite quantum electrodynamics. In this theory the insertion of $\overline{\psi}\psi$, the composite mass operator, into the electron propagator satisfies a homogeneous Bethe-Salpeter equation whose solution is conformally invariant at short distances with an anomalous dimension, $\gamma_{\overline{\psi}\psi}(\alpha)$. To determine whether the theory is forced to actually choose a nontrivial solution to this homogeneous equation we calculate the effective potential, $V(\langle\overline{\psi}\psi\rangle)$, using the dressed scalar vertex as input. We find that the infrared divergences of the theory cause the effective potential to develop a degenerate minimum away from the origin in classical field space. Thus dynamical γ_5 symmetry breaking takes place with the electron acquiring a mass $m \sim \langle\overline{\psi}\psi\rangle$. It is thought that this may be a general mechanism for generating masses in an otherwise conformal-invariant theory.

I. INTRODUCTION

This paper is concerned with the possibility that dynamical γ_5 symmetry breaking is the agency for introducing a scale into particle physics. In discussing theories in which all of the mass is to be dynamical we must start off with an underlying massless theory with dimensionless couplings. Such theories will exhibit conformal invariance with anomalous dimensions at short distances provided there is a renormalization-group fixed point.[1-3] We restrict ourselves to such theories only in this paper. These theories are generally regarded as being off-shell theories only without a sensible mass-shell limit. Thus the good ultraviolet limit is accompanied by a bad infrared limit. In perturbation theory we avoid but do not solve the infrared problem by renormalizing off the mass shell. Eventually, however, we have to sum the perturbation series and go to the mass shell, at which point we then have to face the infrared problem. The main point of this paper is that this infrared problem is then solved by dynamical γ_5 symmetry breaking, so that the fermions in the theory acquire masses by translating to the new vacuum. In this approach Wilson's skeleton theory[4] will be an exactly conformal-invariant renormalizable theory with either anomalous or canonical dimensions at short distances, depending on whether the ultraviolet-stable fixed point is nontrivial or at the origin; and all of the breaking of conformal invariance is achieved through the γ_5 degeneracy of the vacuum with no soft operators (or dilatons) being needed in the theory. This is the realization of an idea we suggested in a recent publication.[5]

The theory we analyze in detail in this paper is Johnson-Baker-Willey quantum electrodynamics[6-11] (finite QED) which possesses an explicit dynamical-symmetry-breaking solution. These authors have considered the Bethe-Salpeter equation (see Fig. 1)

$$m\tilde{\Gamma}_S(p,p,0)=m_0Z_2+\int\frac{d^4k}{(2\pi)^4}\,m\tilde{\Gamma}_S(k,k,0)i\tilde{S}(k)\times\tilde{K}(p,k,0)i\tilde{S}(k) \qquad (1)$$

for the insertion of the renormalized scalar operator, $\theta=\overline{\psi}\psi$, carrying zero momentum into the electron propagator. In the generalized Landau gauge where Z_2 is finite the electron propagator is canonical and the above equation admits of a solution

$$m\tilde{\Gamma}_S(p,p,0)=C(\alpha)m\left(\frac{-p^2}{m^2}\right)^{\gamma_\theta(\alpha)/2} \qquad (2)$$

for asymptotic p.[12] Thus if γ_θ, the anomalous dimension of θ, is negative the theory has a zero bare mass m_0 (in the limit of infinite cutoff).[13] Equation (1) then becomes a homogeneous bootstrap equation for the renormalized mass operator and admits of a nonvanishing physical mass. This

Neutrino pairing as the origin of parity violation in a chiral flavor theory of weak interactions

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In this paper we study the space-time and internal-symmetry aspects of weak interactions together with their interplay. We propose that the weak interaction be obtained by gauging the strong-interaction chiral flavor group and consider explicitly the group $SU(4)_L \times SU(4)_R \times U(1)$ as a candidate unified gauge theory of weak and electromagnetic interactions. In the symmetry limit the theory is both parity conserving and flavor conserving. The neutrinos in the theory are four-component spinors. We introduce a new dynamical mechanism for obtaining parity violation in weak interactions, namely neutrino pairing, in which we allow pairs of right-handed neutrinos to condense into the vacuum. The group theory associated with this pairing produces maximal parity violation in both the lepton and quark charged-current sectors of the weak interaction and also the conventional Weinberg mixing pattern in the neutral-current sector, while keeping the observed left-handed electron and muon neutrinos massless. In the right-handed sector of the theory, however, the pairing causes the right-handed electron and muon neutrinos to combine into one massive four-component spinor. The unusual phenomenology associated with this new observable massive neutral lepton is studied in detail. Using other representations of the gauge group we also break flavor spontaneously to provide a group-theoretical origin for the Cabibbo angle. Through our approach we can incorporate current algebra and the Gell-Mann–Oakes–Renner Hamiltonian phenomenology into weak-interaction theory. We discuss briefly the implications of our work for constructing grand-unified theories of the strong, electromagnetic, and weak interactions.

I. INTRODUCTION

The Hamiltonian is the generator of time translations and the parity operator generates space reflections. Since time translations and space reflections commute, it would therefore be expected that all interactions conserve parity, which of course is not the case. Quantum field theory provides an answer to the above problem inasmuch as geometrical properties of operators need not apply to the states of the theory. For instance, it is often argued that if the neutrino is only a two-component spinor, then the parity operator takes the neutrino state out of the Hilbert space so that for weak interactions the parity operator and matrix elements of the $[H,P]$ commutator simply do not exist. However, this argument is unsatisfactory because it not only sidesteps the issue of whether or not the neutrino should anyway be a two-component spinor, but because it also fails to explain the origin of (maximal) parity violation in processes which do not involve neutrinos. Further, it does not explain why the parity operator does exist for the other fundamental interactions, giving no criterion for when a parity operator is to exist or not. Thus we need a resolution of the problem which not only explains the fact of parity violation, but also produces the systematics of its observed pattern of violation.

It is now recognized that when a physical system possesses an infinite number of degrees of freedom, cooperative phenomena can produce a ground state which has a lower symmetry than that of the Lagrangian. Thus we are led to consider parity to be a spontaneously broken symmetry with H and P indeed commuting at the level of the Lagrangian, with an asymmetric vacuum producing parity-violating S-matrix elements. Since the Lagrangian is parity conserving it must therefore contain both left- and right-handed neutrinos, so that the neutrinos are four-component spinors. At this stage, though, there is nothing which obliges the neutrinos to be massless either before or after parity has been spontaneously broken. However, since the neutrinos appear to be massless experimentally, we shall not introduce any neutrino mass terms into the Lagrangian by imposing a chiral limit, and shall have to constrain the spontaneous breakdown in such a way that it not generate any neutrino mass terms either. Thus we have to seek a specific and natural symmetry-breaking scheme which will keep the neutrinos massless. A successful theory of parity violation will then be one in which this specific requirement on the symmetry-breaking scheme will be such as to produce the detailed picture of violation that exists, namely, maximal parity violation for the weak-interaction charged currents in both the leptonic and nonleptonic sectors of the theory (i.e., even in processes which do not involve neutrinos) together with the Weinberg mixing pattern of parity violation in the neutral-

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EXACT VACUUM SOLUTION TO CONFORMAL WEYL GRAVITY AND GALACTIC ROTATION CURVES

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ABSTRACT

We present the complete, exact exterior solution for a static, spherically symmetric source in locally conformal invariant Weyl gravity. The solution includes the familiar exterior Schwarzschild solution as a special case and contains an extra gravitational potential term which grows linearly with distance. Our obtained solution provides a potential explanation for observed galactic rotation curves without the need for dark matter. Our solution also has some interesting implications for cosmology.

Subject headings: cosmology — dark matter — galaxies: internal motions — gravitation

I. INTRODUCTION

There is by now a broad consensus in the community that the correct theory of gravity (at least at the classical level) is that based on the Einstein equations, i.e., the equations derived by the variation of the Einstein-Hilbert action $I_{EH} = -(1/16\pi G)\int d^4x(-g)^{1/2}R^{\alpha}{}_{\alpha}$. This consensus has been reached on account of the fact that the standard tests of General Relativity have established that, in the vicinity of the Sun, the geometry which fixes the geodesics is that given by the experimentally tested part of the Schwarzschild line element. The fact that particles should indeed move on geodesics is actually a rather general property of curved space time, resulting from the fact that the energy momentum tensor is covariantly conserved, i.e., that $T^{\mu\nu}{}_{;\nu} = 0$, with this latter property itself being true by virtue of the Bianchi identities. A well known (see, e.g., Eddington 1922, pp. 140–144) but not always appreciated fact is that $T^{\mu\nu}$ will be covariantly conserved for any action which is a coordinate scalar, since for any such action there would always be an appropriate set of Bianchi identities. Thus there is therefore an inherent freedom in the choice of the action for the gravitational field. What makes the choice of the Einstein action unique is the additional (simplifying) requirement that the resulting equations of motion be no higher than second order in the metric. Other choices for the action scalar can then yield field equations different from those of Einstein which could still in principle comply with the standard tests of General Relativity.[2] In this note we present (§ II) the field equations resulting from a highly restrictive prescription for choosing the action, one *based on an invariance principle rather than the demand of second-order equations of motion* and provide their complete, exact, vacuum solution for a spherically symmetric geometry (i.e., the analog of the Schwarzschild solution for this theory). Finally, in § III the solution is discussed and contact with observations is made, suggesting a possible explanation for the observed galactic rotation curves without the need to invoke the existence of dark matter.

II. ACTION AND FIELD EQUATIONS

In analogy with the principle of local gauge invariance which severely restricts the structure of possible Lorentz invariant actions in flat spacetime theories, we impose the quite analogous principle of local conformal invariance as the requisite principle to restrict the choice of action for the gravitational field in curved spacetime. This principle requires the action to remain invariant under any and all local stretchings $g_{\mu\nu}(x) \to \Omega^2(x)g_{\mu\nu}(x)$ of the geometry. The action

$$I_W = -\alpha \int d^4x(-g)^{1/2}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa} = -2\alpha \int d^4x(-g)^{1/2}[R_{\mu\kappa}R^{\mu\kappa} - (1/3)(R^{\alpha}{}_{\alpha})^2], \tag{1}$$

where $C_{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor (Weyl 1918) and α is a purely dimensionless coefficient, is completely locally conformal invariant and represents the *unique* four-dimensional coordinate scalar with the requisite properties.[3] Recently one of us (Mannheim 1988) has reopened the question of the Weyl alternative to Einstein gravity by pointing out that in the Weyl theory it is possible to have a nontrivial de Sitter geometry as a solution even in the absence of any cosmological constant term, with such a term vanishing identically precisely because of the conformal invariance of the theory. Thus, such a theory naturally alleviates the discrepancy between theoretical considerations (which suggest that the cosmological constant should be of the order of the Planck energy density) and observation (which suggests that it is at least 10^{120} times smaller). Under the same principle, the familiar Einstein-Hilbert action with its intrinsic Newton's constant scale is also absent. By then bypassing the Einstein action completely, we differ from other approaches to higher order gravity which try either to incorporate the Einstein term in the higher order action *ab initio* or induce it in some appropriate limit. Moreover, since we have a strictly conformally invariant theory, particle masses can only arise through dynamics (i.e., via the spontaneous breaking of the symmetry of the action), a situation which is now anyway thought

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[2] A specific example is given by the field equations resulting from variation of the action $I = \int d^4x(-g)^{1/2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, which as shown by Eddington (1922) do admit the Schwarzschild line element as a solution (this is not their most general solution though) and hence could pass the standard tests of General Relativity.

[3] We hereafter refer to the theory based on the action I_W of eq. (1) as Weyl gravity; this should not be confused with Weyl geometry, a non-Riemannian geometry in which $g^{\mu\nu}{}_{;\alpha}$ is equal to $b_\alpha g^{\mu\nu}$ rather than zero. Our geometry is indeed Riemannian, i.e., b_α is zero.

Fourth Order Theories Without Ghosts*

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Abstract

Using the Dirac constraint method we show that the pure fourth-order Pais-Uhlenbeck oscillator model is free of observable negative norm states. Even though such ghosts do appear when the fourth order theory is coupled to a second order one, the limit in which the second order action is switched off is found to be a highly singular one in which these states move off shell. Given this result, construction of a fully unitary, renormalizable, gravitational theory based on a purely fourth order action in 4 dimensions now appears feasible.

As a theory conformal gravity is somewhat enigmatic. With it being based on a local invariance (viz. $g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)}g_{\mu\nu}(x)$) and with it possessing a gravitational action (viz. $I_W = -\alpha_g \int d^4x(-g)^{1/2}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa}$ where $C^{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor) whose coupling constant α_g is dimensionless, it has a structure which is remarkably similar to that which underlies the other fundamental interactions. Moreover, not only is the conformal gravitational theory power counting renormalizable in four spacetime dimensions, as a quantum theory it turns out to even be asymptotically free [1], while as a classical theory it has [2,3] the confining $V(r) = -\beta/r + \gamma r/2$ as its exact classical potential, to thus reproduce some of the most desirable properties of non-Abelian gauge theories. And further, through use of this linear potential the theory has been found capable [4] of accounting for the perplexing galactic rotation curves without the need to introduce any galactic dark matter, with the cosmology associated with the theory being found [5] to be free of any flatness, horizon, universe age or cosmological dark matter problem, with it being a cosmology which naturally (i.e. without fine tuning) explains the recently detected cosmic acceleration, and which, through its underlying scale invariance, naturally solves [6] the cosmological constant problem. Despite all of these attractive features, standing on the debit side is a potential ghost problem which could render the theory non-unitary. Specifically, being a fourth order theory it has a $D(k^2) = 1/k^4$ propagator, a propagator which by being rewritten as the $M^2 \rightarrow 0$ limit of $D(k^2, M^2) = [1/k^2 - 1/(k^2 + M^2)]/M^2$ can immediately be seen as

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Dirac quantization of the Pais-Uhlenbeck fourth order oscillator

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As a model, the Pais-Uhlenbeck fourth order oscillator with equation of motion: $(d^4q/dt^4)+(\omega_1^2+\omega_2^2)\times(d^2q/dt^2)+\omega_1^2\omega_2^2q=0$ is a quantum-mechanical prototype of a field theory containing both second and fourth order derivative terms. With its dynamical degrees of freedom obeying constraints due to the presence of higher order time derivatives, the model cannot be quantized canonically. We thus quantize it using the method of Dirac constraints to construct the correct quantum-mechanical Hamiltonian for the system, and find that the Hamiltonian diagonalizes in the positive and negative norm states that are characteristic of higher derivative field theories. However, we also find that the oscillator commutation relations become singular in the $\omega_1\to\omega_2$ limit, a limit which corresponds to a prototype of a pure fourth order theory. Thus the particle content of the $\omega_1=\omega_2$ theory cannot be inferred from that of the $\omega_1\neq\omega_2$ theory; and in fact in the $\omega_1\to\omega_2$ limit we find that all of the $\omega_1\neq\omega_2$ negative norm states move off shell, with the spectrum of asymptotic in and out states of the equal frequency theory being found to be completely devoid of states with either negative energy or negative norm. As a byproduct of our work we find a Pais-Uhlenbeck analog of the zero energy theorem of Boulware, Horowitz, and Strominger, and show how in the equal frequency Pais-Uhlenbeck theory the theorem can be transformed into a positive energy theorem instead.

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I. INTRODUCTION

While attention in physics has by and large concentrated on second order equations of motion, nonetheless, from time to time there has also been some interest in higher derivative theories, with a typical such higher order equation of motion being the scalar field equation of motion

$$(\partial_0^2-\nabla^2)(\partial_0^2-\nabla^2+M^2)\phi(\bar{x},t)=0, \quad (1)$$

a wave equation which is based on both second and fourth order derivatives of the field. With the propagator associated with Eq. (1) being given by

$$D(k^2,M^2)=\frac{1}{k^2(k^2+M^2)}=\frac{1}{M^2}\left[\frac{1}{k^2}-\frac{1}{k^2+M^2}\right] \quad (2)$$

in momentum space, the expectation of canonical reasoning (see e.g., Ref. [1] for a canonical study of theories of such second plus fourth order type) is that a quantization of the theory associated with Eq. (1) would, for $M^2\neq 0$, lead to a $1/k^2-1/(k^2+M^2)$ dipole spectrum consisting of two species of particles, one possessing a positive signature and the other a negative or ghost signature. Indeed, part of the appeal of such propagators is that precisely because of this ghost signature, the propagator has much better behavior in the ultraviolet than a standard second order $1/k^2$ propagator, to thus enable this higher order theory to naturally address renormalization issues such as those associated with elementary particle self energies or with quantum gravitational fluctuations.

While a similar conclusion regarding the presence of ghosts might be anticipated to apply to pure fourth order theories as well [viz. the $M^2=0$ limit of Eq. (1)] [2], as we see from the form of Eq. (2), the $1/M^2$ prefactor multiplying the $1/k^2-1/(k^2+M^2)$ dipole term is singular, and thus is not reliable to infer the structure of the $M^2=0$ spectrum from that associated with that of $M^2\neq 0$. In fact, it is not actually reliable to try to infer the spectrum associated with Eq. (1) via canonical reasoning at all, since even when $M^2\neq 0$, the theory is constrained due to the presence of higher order time derivatives. Thus before drawing any conclusions at all, one must first quantize the theory in a way which fully takes these constraints into account, and only then identify the particle spectrum. To address this issue we shall thus effect a full Dirac constraint quantization [3] of a quantum-mechanical prototype of Eq. (1), something which has not previously been carried out in the literature. Specifically, we shall study a restricted version of Eq. (1) in which we specialize to field configurations of the form $\phi(\bar{x},t)=q(t)e^{i\bar{k}\cdot\bar{x}}$, configurations in which Eq. (1) then reduces to

$$\frac{d^4q}{dt^4}+(\omega_1^2+\omega_2^2)\frac{d^2q}{dt^2}+\omega_1^2\omega_2^2q=0, \quad (3)$$

where

$$\omega_1^2+\omega_2^2=2\bar{k}^2+M^2, \quad \omega_1^2\omega_2^2=\bar{k}^4+\bar{k}^2M^2, \quad (4)$$

with Eq. (4) reducing to the equal frequency $\omega_1=\omega_2$ when $M=0$. As such, for general $\omega_1\neq\omega_2$ Eq. (3) thus serves as a quantum-mechanical prototype of a field theory based on second order plus fourth order derivatives of the field, while becoming a prototype of a pure fourth order theory in the equal frequency limit. In addition to being a model which we

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No-Ghost Theorem for the Fourth-Order Derivative Pais-Uhlenbeck Oscillator Model

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A new realization of the fourth-order derivative Pais-Uhlenbeck oscillator is constructed. This realization possesses no states of negative norm and has a real energy spectrum that is bounded below. The key to this construction is the recognition that in this realization the Hamiltonian is not Dirac Hermitian. However, the Hamiltonian is symmetric under combined space reflection P and time reversal T. The Hilbert space that is appropriate for this PT-symmetric Hamiltonian is identified and it is found to have a positive-definite inner product. Furthermore, the time-evolution operator is unitary.

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It has long been thought that field theories based on equations of motion higher than second order are unacceptable because they possess states, known as *ghosts*, which have nonpositive norm. The purpose of this Letter is to show that this is not necessarily so, and thereby to regenerate interest in higher-order quantum field theories. Higher-order theories appear in a variety of contexts [1] and are potentially of great interest.

To explain the issues involved, we review the Lee model. The model was proposed in 1954 as a trilinearly coupled quantum field theory in which the renormalization program can be carried out in closed form [2]. However, just one year later it was argued that this theory has a ghost state [3]. Specifically, a ghost appears in the Lee model when the renormalized coupling constant exceeds a critical value. Above this critical value, the Lee-model Hamiltonian becomes non-Hermitian in the Dirac sense because its trilinear interaction term acquires an imaginary coefficient. (Dirac-Hermitian conjugation is combined matrix transposition and complex conjugation.) In the non-Hermitian phase of the Lee model a state of negative Dirac norm emerges.

For the past half century, there have been many attempts to make sense of the Lee model as a valid quantum theory (starting as early as [4]), but it was not until 2005 that it was shown that it is possible to formulate the theory without a ghost [5]. The solution to the Lee-model-ghost problem is that when the coupling constant exceeds its critical value, the Hamiltonian transits from being Dirac Hermitian to being PT symmetric, i.e., symmetric under combined parity reflection and time reversal. In the sector in which the ghost appears the PT symmetry is *unbroken*; that is, all energy eigenvalues are real. For any non-Hermitian, PT-symmetric Hamiltonian in such a phase, one should not use the Dirac norm. Rather, one should introduce an alternate inner product [6–9]. With respect to the new inner product appropriate for the PT-symmetric phase of the Lee model, the Hamiltonian becomes self-adjoint and its ghost state is reinterpreted as an ordinary quantum state with positive PT norm. This same procedure has been applied to other problematic models [10].

The purpose of this Letter is to show that this same prescription can be implemented in the fourth-order-derivative Pais-Uhlenbeck (PU) oscillator model, the prototypical higher-derivative quantum field theory. We construct here a realization in which the Hamiltonian has an unbroken PT symmetry and the negative Dirac-norm states that are thought to arise are really ordinary quantum states having positive PT norm.

The action of the PU model is acceleration dependent:

$$I_{\mathrm{PU}} = \frac{\gamma}{2}\int dt[\ddot{z}^2 - (\omega_1^2 + \omega_2^2)\dot{z}^2 + \omega_1^2\omega_2^2 z^2], \tag{1}$$

where γ, ω_1, and ω_2 are all positive constants, and without loss of generality we take $\omega_1 \geq \omega_2$ [11]. This model represents two oscillators coupled by a fourth-order equation of motion: $d^4z/dt^4 + (\omega_1^2 + \omega_2^2)d^2z/dt^2 + \omega_1^2\omega_2^2 z = 0$ [12]. With $\dot{z}$ serving as the canonical conjugate of both z and $\ddot{z}$, the system is constrained and its Hamiltonian can be found by the method of Dirac constraints. To this end, in place of $\dot{z}$ we introduce a new dynamical variable x (with corresponding conjugate p_x), and via the Dirac method we construct the Hamiltonian [13]

$$H_{\mathrm{PU}} = \frac{p_x^2}{2\gamma} + p_z x + \frac{\gamma}{2}(\omega_1^2 + \omega_2^2)x^2 - \frac{\gamma}{2}\omega_1^2\omega_2^2 z^2. \tag{2}$$

This Hamiltonian depends on two coordinates x and z, and their canonical conjugates, p_x and p_z. The Poisson-bracket algebra of the five operators x, p_x, z, p_z, and H is closed, with nonzero brackets in the x, p_x, z, p_z sector being $\{x, p_x\} = 1$ and $\{z, p_z\} = 1$. This construction is independent of the classical equations of motion and thus holds for both stationary and nonstationary classical paths. Thus, we can use it to quantize the model, with nonzero commutators being $[x, p_x] = i$ and $[z, p_z] = i$.

Impact of a Global Quadratic Potential on Galactic Rotation Curves

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We present a conformal gravity fit to the 20 largest of a sample of 110 spiral galaxies. We identify the presence of a universal quadratic potential $V_\kappa(r) = -\kappa c^2 r^2/2$ with $\kappa = 9.54 \times 10^{-54}\text{cm}^{-2}$ induced by cosmic inhomogeneities. When $V_\kappa(r)$ is taken in conjunction with both a universal linear potential $V_{\gamma_0}(r) = \gamma_0 c^2 r/2$ with $\gamma_0 = 3.06 \times 10^{-30}\text{cm}^{-1}$ generated by the homogeneous cosmic background and the contribution generated by the local luminous matter in galaxies, the theory then accounts for the rotation curve systematics observed in the entire 110 galaxies, without the need for any dark matter whatsoever. Our study suggests that using dark matter may be nothing more than an attempt to describe global effects in purely local galactic terms. With $V_\kappa(r)$ being negative, galaxies can only support bound orbits up to distances of order $\gamma_0/\kappa = 100$kpc, with global physics imposing a limit on the size of galaxies.

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I. Introduction.—At the present time it is widely believed that on scales much larger than solar-system-sized ones astrophysical and cosmological phenomena are controlled by dark matter and dark energy, with luminous matter being only a minor contributor. However, given the lack to date of either direct detection of dark matter particles or of a solution to the cosmological constant problem, a few authors (see, e.g., [1] for a recent review) have ventured to suggest that the standard dark matter and dark energy picture may be incorrect, and that one instead needs to modify the standard Newton-Einstein gravitational theory that leads to that picture in the first place. In this Letter we study one specific alternative to Einstein gravity that has been advanced, namely, conformal gravity. We report here on the results of a conformal gravity study of the instructive 20 largest of a full sample of 110 galaxies, all of whose rotation curves we have been able to fit without the need for any dark matter whatsoever.

In seeking an alternative to Einstein gravity that is to address both the dark matter and dark energy problems, our strategy is to seek some alternate, equally metric-based theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of Einstein gravity, that yields a geometry that is described by the Ricci-flat Schwarzschild metric on solar-system-sized distance scales while departing from it on larger scales where the dark matter problem is first encountered, and that has a symmetry that can control the cosmological constant Λ. All of these criteria are met in the conformal gravity theory (see, e.g., [1]) that was first developed by Weyl. Specifically, as well as coordinate invariance, in addition one requires that the action be left invariant under local conformal transformations of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Given this requirement, the gravitational action is then uniquely prescribed to be of the form $I_W = -2\alpha_g \int d^4x(-g)^{1/2}[R_{\mu\kappa}R^{\mu\kappa} - (1/3)\times (R^\alpha{}_\alpha)^2]$ where α_g is a dimensionless gravitational coupling constant. With the conformal symmetry forbidding the presence of any fundamental Λ term in I_W, conformal gravity has a control on Λ that is not possessed by Einstein gravity, and through this control conformal gravity is then able to solve the cosmological constant problem [2]. In addition, the conformal gravity equations of motion are given by [1]

$$4\alpha_g W^{\mu\nu} = T^{\mu\nu} \tag{1}$$

where $W^{\mu\nu}$ is a derivative function of $R^{\mu\nu}$. With $W^{\mu\nu}$ vanishing when $R^{\mu\nu}$ vanishes [1], Schwarzschild is thus a vacuum solution to conformal gravity, just as required [3].

II. Universal potentials from the rest of the Universe.—Since $W^{\mu\nu}$ is a derivative function of $R^{\mu\nu}$, it could potentially vanish even if the geometry is not Ricci flat, and the conformal theory could thus have non-Schwarzschild vacuum solutions as well. To identify such solutions, Mannheim and Kazanas solved for the metric associated with a static, spherically symmetric source, to find [4] that due to the underlying conformal symmetry one could bring the exact, all-order line element to the form $ds^2 = -B(r)dt^2 + dr^2/B(r) + r^2 d\Omega_2$. And with $3(W^0{}_0 - W^r{}_r)/B(r)$ then evaluating to $\nabla^4 B(r)$, the metric coefficient $B(r)$ is found to obey the remarkably simple and exact fourth-order derivative equation

$$\nabla^4 B(r) = f(r) \tag{2}$$

where $f(r) = 3(T^0{}_0 - T^r{}_r)/4\alpha_g B(r)$. For a local source of radius r_0 embedded in an empty vacuum (2) possesses an exterior solution of the form

$$B(r > r_0) = 1 - 2\beta/r + \gamma r. \tag{3}$$

Through the γr term the conformal gravity metric thus departs from the exterior Schwarzschild metric at large r alone, just as we want.

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Antilinearity rather than Hermiticity as a guiding principle for quantum theory

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Abstract

Currently there is much interest in Hamiltonians that are not Hermitian but instead possess an antilinear *PT* symmetry. Here we seek to put such *PT* symmetric theories into as general a context as possible. After providing a brief overview of the *PT* symmetry program, we show that having an antilinear symmetry that acts on a well-defined Hilbert space is the most general condition that one can impose on a quantum theory for which one can have a well-defined inner product that is time independent, have a Hamiltonian that is self-adjoint, and have energy eigenvalues that are all real. For each of these properties Hermiticity is only a sufficient condition but not a necessary one, with Hermiticity thus being the special case in which the Hamiltonian has both antilinearity and Hermiticity. As well as being the necessary condition for the reality of energy eigenvalues, antilinearity in addition allows for the physically interesting cases of manifestly non-Hermitian but nonetheless self-adjoint Hamiltonians that have energy eigenvalues that appear in complex conjugate pairs, or that are Jordan block and cannot be diagonalized at all. We show that one can extend these ideas to quantum field theory, with the dual requirements of the existence of time independent inner products and invariance under complex Lorentz transformations forcing the antilinear symmetry to uniquely be *CPT*. We thus extend the *CPT* theorem to non-Hermitian Hamiltonians. For theories that are separately charge conjugation invariant, *PT* symmetry then follows, with the case for the physical relevance of the *PT*-symmetry program thus being advanced. While *CPT* symmetry can be defined at the classical level for every classical path in a path integral quantization procedure, in contrast, in such a path integral there is no reference at all to the Hermiticity of the Hamiltonian or the quantum Hilbert space on which it acts, as they are strictly quantum-mechanical concepts that can only be defined after the path integral quantization has been performed and the quantum Hilbert space has been constructed. *CPT* symmetry thus goes beyond Hermiticity and has primacy over it, with our work raising the question of how Hermiticity ever

Central Themes of my Research

Already in graduate school I was puzzled by two issues. 1. If time translations and space reflections commute how can a Hamiltonian not conserve parity, so how could $[H,P]$ not be equal to zero. 2. Two-component right-handed and two-component left-handed Weyl spinors are not connectable by Lorentz transformations, with the four-component Dirac spinor thus being reducible under the Lorentz group. So how could the fundamental building blocks of matter (the four-component Dirac spinors) not be in the fundamental representation of the fundamental group of nature. This question ties in with the question of the origin of mass, as that involves Dirac not Weyl spinors. The resolution of these issues has been central to my research.

There was a canonical answer to the parity question, namely that right-handed neutrinos did not exist, and thus the action of the $[H,p]$ commutator on a left-handed neutrino was not defined. But there are parity-violating weak interactions that do not involve neutrinos, such $\Lambda \rightarrow p + \pi^-$. And anyway, who decided that right-handed neutrinos do not exist. Moreover, if right-handed neutrinos did exist they had to have a very high mass so as to have escaped detection. Thus again we are lead to the issue of the origin of mass.

When I joined UConn on 1979 I was well launched into these questions, with my interview talk being entitled "Phase Transitions in Many-Body Theory and Particle Physics" Over the course of my years at UConn I was able to address these questions, and it lead me in directions that I could not have imagined in 1979 such as PT symmetry, the dark matter and dark energy problems, and quantum gravity.

The central theme of all this work is the application of many-body ideas to particle physics and then to cosmology. In many-body theory a system can possess collective properties that are not possessed by the individual particles that make up the system. Temperature is a familiar example, but more interesting is the existence of a long range order parameter, such as the spontaneous magnetization that occurs in a ferromagnet below the Curie-Weiss temperature. In such situations the solutions to the equations of motion have lower symmetry than the equations of motion themselves.

In a superconductor there are bound Cooper pairs of electrons and they produce the gap parameter that provides for a mass shift to the electrons, i.e., it provides for some of the total mass. It was suggested by Nambu that all of the mass of elementary particles could be generated this way. Now it was known through the work of Kadanoff and Wilson on the renormalization group that one had critical scaling at the phase transition temperature, with correlation functions changing from exponential fall off to a power fall off with anomalous dimensions. However, at the critical point the order parameter is zero. Thus critical scaling and spontaneous mass generation were disconnected.

So I set out to try to connect them. In work done in the 1960s (before Kadanoff and Wilson) Johnson, Baker and Willey had found that the bare mass of the electron would be zero if the electron propagator scaled as p^γ with $\gamma < 0$, with the ultraviolet behavior of the theory then being under control. But would the physical mass be nonzero. Now as γ becomes more negative the theory becomes more convergent in the ultraviolet, and thus becomes more divergent in the infrared. Then in 1974 I showed that if γ drops all the way to $\gamma = -1$ then the infrared divergences would become so severe that the theory would be forced into a spontaneously broken vacuum in which there would be long range order with nonzero order parameter $\langle\Omega|\bar{\psi}\psi|\Omega\rangle \neq 0$. Thus I connect scaling with spontaneous symmetry breaking and mass generation.

What I realized at the time was that with this critical scaling one had conformal symmetry. But it took me many years to realize its implications, with things only coming to fruition once tenure at UConn provided me with the opportunity to work on these kind of long-term issues.

Just as I was moving to UConn in 1979 I found the answer to the parity issue, namely $[H, P]$ is zero after all and parity is spontaneously broken. But how? Should be something like Cooper pairing, but a pair of electrons cannot acquire a nonzero vacuum expectation value because of charge conservation. But a pair of neutrinos can. So try $\langle\Omega|\psi\psi|\Omega\rangle$. For this combination both neutrinos have to have the same handedness, so either both right-handed or both left-handed. But either way parity is broken. So take the right-handed pair expectation value to be nonzero. Then right-handed neutrinos can have a high mass, and can hence avoid detection.

Getting into General Relativity

In 1980 Alan Guth developed the theory of early universe inflation, and this provided a bridge between elementary particle physics and cosmology. So I needed to study General Relativity (it was not taught when I was in graduate school), and asked the UConn Physics Department if I could teach it. And so I gradually started to understand the subject.

Conformal Gravity

In 1987 I got a National Academy of Sciences Fellowship to spend a year at the NASA Goddard Space Flight Center in Greenbelt Maryland. While there I worked with Demos Kazanas, a Goddard astrophysicist. His knowledge of astrophysics (he had also developed inflation before Guth) complemented my knowledge of particle physics, and we were able to collaborate very easily.

At that time one of the big open questions in physics was the cosmological constant problem (now called the dark energy problem), namely how the cosmological constant could be 60 orders of magnitude smaller than its particle physics expectation. While at Goddard I realized that if gravity had a conformal structure the cosmological constant would be zero. And thus I could tie the mass scale of the cosmological constant to the mass generation of elementary particles that I had studied in the 1970s.

We wrote down a gravity theory that would have a conformal structure, namely conformal gravity with its gravitational action $\int d^4x(-g)^{1/2}C_{\lambda\mu\nu\tau}C^{\lambda\mu\nu\tau}$, where $C_{\lambda\mu\nu\tau}$ is the conformal Weyl tensor. The conformal symmetry forbids the presence of any cosmological constant term of the form $\int d^4x(-g)^{1/2}\Lambda$ (which is good) but also forbids the standard Einstein-Hilbert action $\int d^4x(-g)^{1/2}(1/8\pi G)R^{\alpha}{}_{\alpha}$ (which is bad?). However, exact conformal invariance excludes particle mass scales as well, and thus to generate them the conformal symmetry must be spontaneously broken, just like my study of the electron mass in quantum electrodynamics.

So without the Einstein-Hilbert action our immediate and pressing concern was what happens to Newton's Law of Gravity, the absolute sine qua non for any gravitational theory. So Demos and I realized that if the conformal theory was going to have any relevance at all we would have to solve the theory to see if we could recover Newton's $1/r$ gravitational potential. To do this we had to deal with some very complicated, fourth-order derivative, equations, and it took us all of six months just to determine what the equations even looked like. At that time there were no software packages for anything other than Einstein gravity, and so we had to develop our own, which we did using Macsyma.

2. STATIC STANDARD COORDINATES

In this case the line element is given by

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega ,$$

and the relevant components of $W^{\mu\nu}$ are denoted by SS^{rr} and SS^{tt} (in this case SS^{rt} vanishes identically). The forms which below are the ones which we used in Paper I.

$$SS^{rr} =$$

$$\begin{aligned}
&(8 A^2 B^2 B_r B_{rrr} r^4 - 4 A^2 B^2 (B_{rr})^2 r^4 - 12 A^2 B (B_r)^2 B_{rr} r^4 \\
&- 8 A A_r B^2 B_r B_{rr} r^4 + 7 A^2 (B_r)^4 r^4 + 6 A A_r B (B_r)^3 r^4 \\
&- 4 A A_{rr} B^2 (B_r)^2 r^4 + 7 (A_r)^2 B^2 (B_r)^2 r^4 - 16 A^2 B^3 B_{rrr} r^3 \\
&+ 48 A^2 B^2 B_r B_{rr} r^3 + 16 A A_r B^3 B_{rr} r^3 - 20 A^2 B (B_r)^3 r^3 \\
&- 16 A A_r B^2 (B_r)^2 r^3 + 16 A A_{rr} B^3 B_r r^3 - 28 (A_r)^2 B^3 B_r r^3 \\
&- 32 A^2 B^3 B_{rr} r^2 - 4 A^2 B^2 (B_r)^2 r^2 + 8 A A_r B^3 B_r r^2 - 16 A A_{rr} B^4 r^2 \\
&+ 28 (A_r)^2 B^4 r^2 + 32 A^2 B^3 B_r r + 16 A^4 B^4 - 16 A^2 B^4)/(48 A^5 B^4 r^4)
\end{aligned}$$

After six months of wrong turns we finally found the equations, and then it took us just a couple of hours to solve them. When we found the solution we found that we indeed got the $1/r$ Newton potential that we wanted, but we found that it was accompanied by a second potential, a linear one, that we absolutely had not expected. The full potential for a star thus took the form $V^*(r) = -\beta^*/r + \gamma^* r/2$.

We also realized two other things: 1. Since we had obtained Newton's Law without Einstein, Einstein gravity was only **SUFFICIENT** for Newton but not necessary. 2. If the parameter γ^* is small enough we would recover Newton on solar system distance scales, but see departures from it on larger scales such as galactic. Then with one potential falling and the other growing their average could be flat, and so we could potentially explain the troublesome flat galactic rotation curves without dark matter.

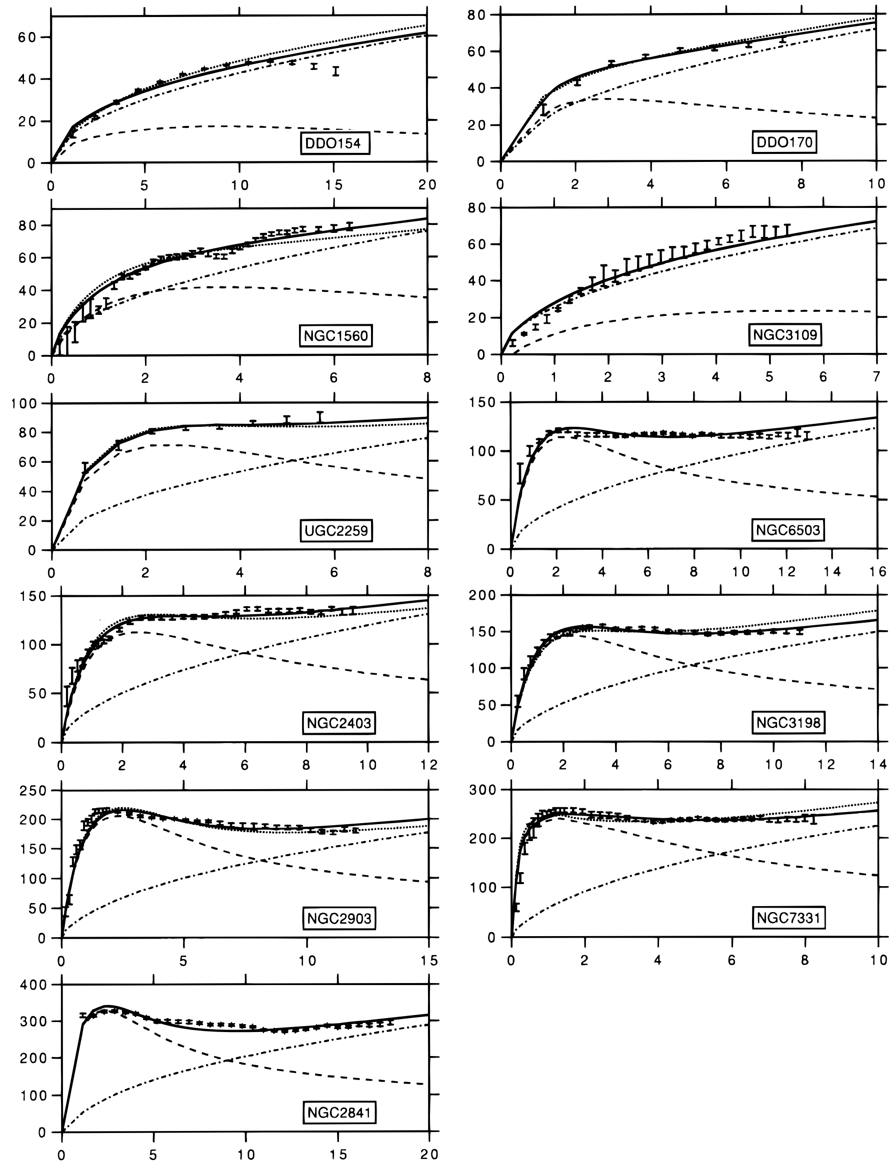

FIG. 1.—Predicted rotational velocity curves associated with conformal gravity for each of the 11 galaxies in the sample. In each graph the bars show the data points with their quoted errors; the full curve shows the overall (adopted distance adjusted) theoretical velocity prediction (in km s^{-1}) as a function of distance from the center of each galaxy (in units of R/R_0, where each time R_0 is each particular galaxy's own optical disk scale length), while the dashed and dash-dotted curves show the velocities that the Newtonian and the linear potentials would produce separately. The dotted curves show the total velocities that would be produced without any adopted distance modification. No dark matter is assumed.

But does it actually work in practice. For a galaxy with N^* stars the potential would be of the form

$$V_{gal}(r) = -N^*\beta^*/r + N^*\gamma^* r/2,$$

and so I myself tried a fit to four representative galaxies, and it worked perfectly except for one problem: I found that rather than γ^* being constant the quantity $N^*\gamma^*$ came out constant even though galaxies had differing N^*. Thus I had to vary γ^* from one galaxy to the next with $\gamma^* \sim 1/N^*$, which could not possibly be correct if the gravity law is to be universal with the γ^* of a star being constant as we varied from one star in a galaxy to the next.

To see how broad this unacceptable conclusion might be, together with a summer REU student, Jan Kmetko, we studied the full set of 11 galaxies that was available at the time. And we found that the problem persisted with the fits being fine but with $N^*\gamma^*$ needing to be constant rather than γ^* itself. Moreover, similar results were obtained by Carlson and Lowenstein at exactly the same time as we obtained ours. However, we had unearthed a possible clue, the numerical value obtained for $N^*\gamma^*$ came out to be of order the inverse of the Hubble radius, a cosmologically relevant scale. But how could cosmology possibly be relevant within individual galaxies?

I sensed that I was on the right track but I did not know what to do. In all I spent seven years trying to understand what was going on. And then finally it hit me. I had being doing things wrong. I had been doing what we always do in Newtonian gravity, in electrodynamics, and in the Schrödinger equation: namely, take a local source and integrate over the potentials produced by the components of the local source. This had been ingrained in me. However, that only works if potentials are local and fall off at large distance. And a linear potential is not local, it grows with distance. Thus I need to include the contribution of material outside the galaxy of interest as those sources have linear potentials of their own. Thus I essentially need to consider the effect on an individual galaxy of the entire rest of the universe. Viola: cosmology.

Now in our original paper Demos and I had shown that if we wrote an expanding Robertson-Walker cosmology in a static coordinate system it looked just like none other than a linear potential. Thus there are **TWO** linear potentials, not one, one being local and the other being global. Thus the potential is now

$$V_{gal}(r) = -N^*\beta^*/r + N^*\gamma^* r/2 + \gamma_0 r/2.$$

And with $\beta^* = 1.48 \times 10^5$ cm, $\gamma^* = 5.42 \times 10^{-41}\text{cm}^{-1}$, $\gamma_0 = 3.06 \times 10^{-30}\text{cm}^{-1}$, this one worked very well on the 11 galaxy sample, and the earlier $V_{gal}(r) = -N^*\beta^*/r + N^*\gamma^* r/2$ fits worked because for bright spirals numerically $N^*\gamma^* \approx \gamma_0$

Encouraged by this result I looked at the accelerating universe data and found that I could fit the data without any fine tuning as the cosmological constant was under control and I did not need any cosmological dark matter. I obtained the following luminosity-redshift formula for the Hubble plot

$$d_L = -\frac{c}{H_0}\frac{(1+z)^2}{q_0}\left(1-\left[1+q_0-\frac{q_0}{(1+z^2)}\right]^{1/2}\right). \tag{1}$$

with the current era deceleration parameter being constrained to $-1 \leq q_0 \leq 0$, i.e., automatically accelerating. Again, the fits were very good.

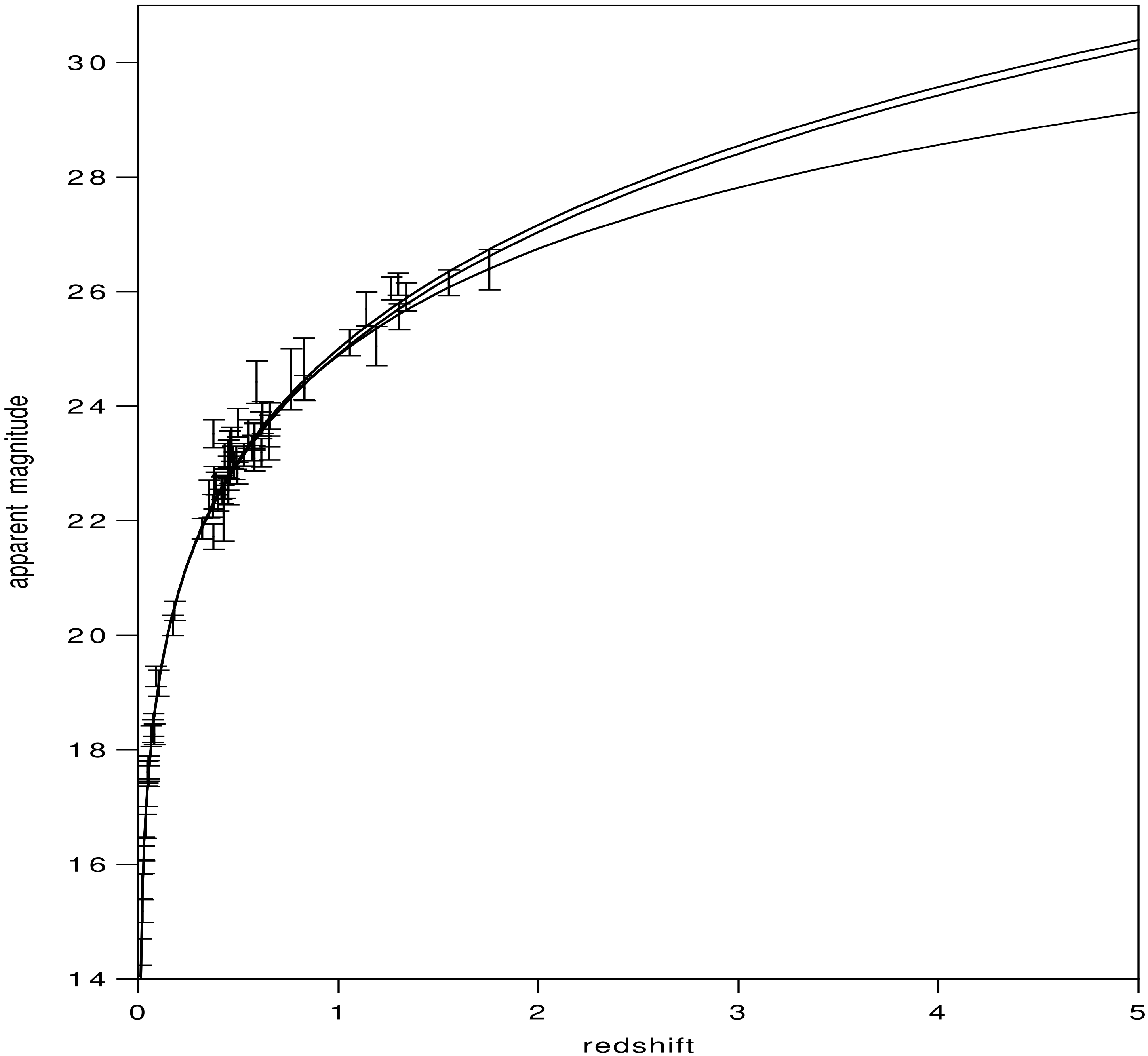

Figure 1: **Hubble plot expectations for $q_0 = -0.37$ (highest curve) and $q_0 = 0$ (middle curve) conformal gravity and for $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard gravity (lowest curve).**

While I could get flat galactic rotation curves by an interplay between falling and raising potentials, eventually the rising one would win and the flat curves should start to rise. With new galactic rotation curve data coming on line, in 2011 James O'Brien and I looked to see if there would be any sign of this rise. James found 111 galaxies, and the data showed no sign whatsoever of any rise. We looked to see where the problem was and found 21 galaxies that went out far enough in distance that in 200 or so data points the rise should have been seen, but there was no sign of any rise at all.

So I had to go back to the drawing board yet again. In obtaining the $\gamma_0 r/2$ potential term I had only included the homogeneous background Hubble flow. However, there are inhomogeneities in the Hubble flow such as clusters of galaxies, and they also put out linear potentials. Their effect led to one more potential term, a quadratic one, to give the full potential the form

$$V_{gal}(r) = -N^*\beta^*/r + N^*\gamma^*r/2 + \gamma_0 r/2 - \kappa r^2$$

And with just one number, viz. $\kappa = 9.54 \times 10^{-54}\text{cm}^{-2}$, we found that we could then fit the entire 200 troublesome points. And a prediction that rotation curves would rise is replaced by one that they would fall.

This study shows the power of having a big data set. And remarkably the numerical values found for γ_0 and κ are respectively a cosmological and a cluster of galaxies scale just as they should be.

From the perspective of a local $1/r$ Newtonian potential the fact that the measured velocities exceed the luminous Newtonian expectation is described as the missing mass problem, with undetected or dark matter within the galaxies themselves being needed in order to be able to account for the shortfall. (In dark matter theory for the 111 galaxy sample one needs 222 more free parameters than in the conformal case.)

From the perspective of conformal gravity the shortfall is explained by the rest of the visible mass in the universe. The missing mass is thus not missing at all, it is the rest of the visible universe and **it has been hiding in plain sight all along**.

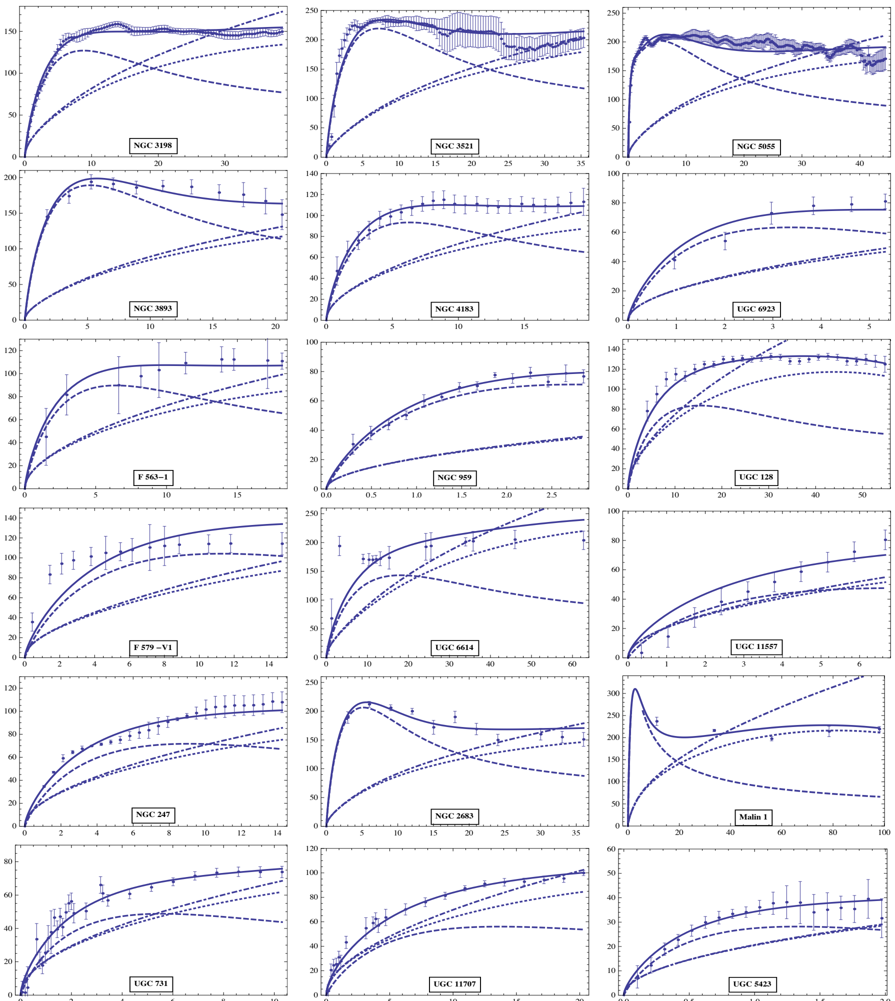

Figure 1. Conformal gravity fitting to the rotational velocities (in km sec^{-1}) of the selected 18 galaxy sample with their quoted errors as plotted as a function of radial distance (in kpc). For each galaxy we have exhibited the contribution due to the luminous Newtonian term alone (dashed curve), the contribution from the two linear terms alone (dot-dashed curve), the contribution from the two linear terms and the quadratic terms combined (dotted curve), with the full curve showing the total contribution. No dark matter is assumed.

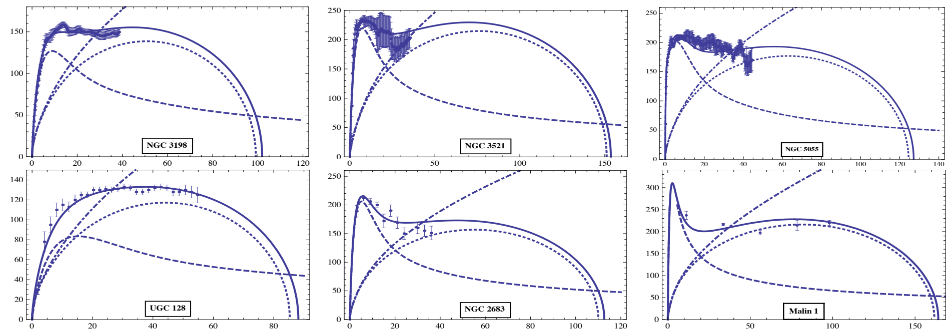

Figure 2. Extended distance predictions for NGC 3198, NGC 3521, NGC 5055, UGC 128, NGC 2683 and Malin 1. The curves are the same as in Figure 1.

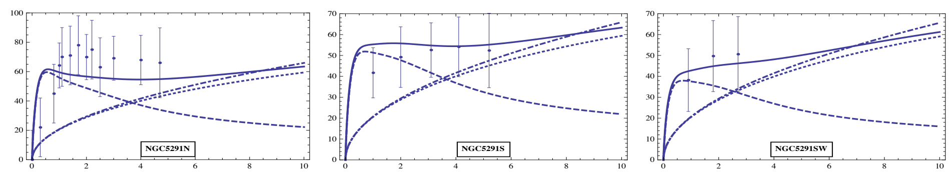

Figure 3. Rotation curves for the tidal dwarf galaxies NGC 5291N, NGC 5291S, and NGC 5291SW, all with inclination 45°. The curves are the same as in Figure 1.

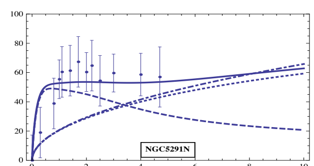

Figure 4. Rotation curve for the tidal dwarf galaxy NGC 5291N with inclination 55°. The curves are the same as in Figure 1.

Table 1. Properties of the Selected 18 Galaxy Sample

Galaxy	D (Mpc)	$L_{\rm B}$ ($10^{10} L_{\odot}^{\rm B}$)	$(R_0)_{\rm disk}$ (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10} M_{\odot}$)	$M_{\rm disk}$ ($10^{10} M_{\odot}$)	$(M/L)_{\rm stars}$ ($M_{\odot}/L_{\odot}^{\rm B}$)	$(v^2/c^2R)_{\rm last}$ (10^{-30} cm^{-1})	Data Sources v	L	R_0	HI
NGC 3198	14.1	3.24	4.0	38.6	1.06	3.64	1.12	2.09	[23]	[24]	[25]	[24]
NGC 3521	12.2	4.77	3.3	35.3	1.03	9.25	1.94	4.21	[23]	[24]	[26]	[24]
NGC 5055	9.2	3.62	2.9	44.4	0.76	6.04	1.87	2.36	[23]	[24]	[26]	[24]
NGC 3893	18.1	2.93	2.4	20.5	0.59	5.00	1.71	3.85	[27]	[28]	[29]	[28]
NGC 4183	16.7	1.04	2.9	19.5	0.30	1.43	1.38	2.36	[27]	[28]	[29]	[28]
UGC 6923	18.0	0.30	1.5	5.3	0.08	0.35	1.18	4.43	[27]	[28]	[30]	[28]
F563-1	46.8	0.14	2.9	18.2	0.29	1.35	9.65	2.44	[31]	[32]	[32]	[33]
NGC 959	13.5	0.33	1.3	2.9	0.05	0.37	1.11	7.43	[34]	[35]	[36]	[35]
UGC 128	64.6	0.60	6.9	54.8	0.73	2.75	4.60	1.03	[37]	[32]	[38]	[38]
F579-V1	86.9	0.56	5.2	14.7	0.21	3.33	5.98	3.18	[31]	[33]	[32]	[33]
UGC 6614	86.2	2.11	8.2	62.7	2.07	9.70	4.60	2.39	[31]	[39]	[38]	[38]
UGC 11557	23.7	1.81	3.0	6.7	0.25	0.37	0.20	3.49	[31]	[39]	[40]	[40]
NGC 247	3.6	0.51	4.2	14.3	0.16	1.25	2.43	2.94	[41]	[42]	[42]	[41]
NGC 2683	10.2	1.88	2.4	36.0	0.15	6.03	3.20	2.28	[43]	[43]	[44]	[45]
Malin 1	338.5	7.91	84.2	98.0	5.40	1.00	1.32	1.77	[46]	[46]	[47]	[46]
UGC 731	11.8	0.07	2.4	10.3	0.16	0.32	4.63	1.91	[48]	[49]	[14]	[40]
UGC 11707	21.5	0.11	5.8	20.3	0.68	0.99	8.76	1.77	[48]	[49]	[14]	[40]
UGC 5423	7.1	0.01	0.6	2.0	0.01	0.03	2.01	1.82	[50]	[24]	[50]	[24]

Table 2. Properties of the 3 Tidal Dwarf Galaxies

Galaxy	D (Mpc)	$L_{\rm B}$ (10^8 L$_{\odot}^{\rm B}$)	i $\circ$	$(R_0)_{\rm gas}$ (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm gas}^{\rm tot}$ ($10^8 M_{\odot}$)	$M_{\rm disk}$ ($10^8 M_{\odot}$)	$(M/L)_{\rm disk}$ ($M_{\odot}/L_{\odot}^{\rm B}$)	$(v^2/c^2R)_{\rm last}$ (10^{-30}cm^{-1})
NGC 5291N	62.0	9.5	45	0.8	4.7	7.7	3.4	0.35	5.7
NGC 5291S	62.0	10.7	45	0.8	5.2	8.6	2.1	0.20	1.9
NGC 5291SW	62.0	5.7	45	0.8	2.7	4.6	1.1	0.20	3.4
NGC 5291N	62.0	9.5	55	0.8	4.7	7.7	1.9	0.20	4.3

Cosmological Fluctuations

The next challenge for the conformal gravity theory is to study the fluctuations around the cosmic microwave background, not just to do it in and and of itself, but also to see to if the scale associated with κ is indeed imprinted on the fluctuations. This work is ongoing being done with my students Matthew Phelps, Asanka Amarasinghe, Tianye Liu and Daniel Norman. To show the nature of the challenge, the need for software packages and the extraordinary simplifications provided by conformal symmetry I present one calculation.

$$
\begin{aligned}
\delta W_{\mu\nu} = {}& \Omega^{-5}\partial_\alpha\partial_\nu\partial^\alpha\Omega\partial_\beta K_\mu{}^\beta + \Omega^{-5}\partial_\alpha\partial_\mu\partial^\alpha\Omega\partial_\beta K_\nu{}^\beta + 2\Omega^{-5}\partial^\alpha\partial_\nu\Omega\partial_\beta\partial_\alpha K_\mu{}^\beta \\
&+ 2\Omega^{-5}\partial^\alpha\partial_\mu\Omega\partial_\beta\partial_\alpha K_\nu{}^\beta + 2\Omega^{-5}\partial^\alpha\Omega\partial_\beta\partial_\alpha\partial_\mu K_\nu{}^\beta + 2\Omega^{-5}\partial^\alpha\Omega\partial_\beta\partial_\alpha\partial_\nu K_\mu{}^\beta \\
&+ \tfrac{1}{3}\Omega^{-4}\partial_\beta\partial_\alpha\partial_\nu\partial_\mu K^{\alpha\beta} - \tfrac{2}{3}K^{\alpha\beta}\Omega^{-5}\partial_\beta\partial_\alpha\partial_\nu\partial_\mu\Omega + \Omega^{-5}\partial^\alpha\partial_\nu\Omega\partial_\beta\partial^\beta K_{\mu\alpha} \\
&- 2\Omega^{-5}\partial_\alpha\partial^\alpha\Omega\partial_\beta\partial^\beta K_{\mu\nu} + 6\Omega^{-6}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta\partial^\beta K_{\mu\nu} + \Omega^{-5}\partial^\alpha\partial_\mu\Omega\partial_\beta\partial^\beta K_{\nu\alpha} \\
&+ 3K_{\mu\nu}\Omega^{-6}\partial_\alpha\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega + 12\Omega^{-6}\partial_\alpha K_{\mu\nu}\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega - 24K_{\mu\nu}\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega \\
&- 4\Omega^{-5}\partial^\alpha\Omega\partial_\beta\partial^\beta\partial_\alpha K_{\mu\nu} + 12K_{\mu\nu}\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial^\beta\partial_\alpha\Omega + \tfrac{1}{2}\Omega^{-4}\partial_\beta\partial^\beta\partial_\alpha\partial^\alpha K_{\mu\nu} \\
&- K_{\mu\nu}\Omega^{-5}\partial_\beta\partial^\beta\partial_\alpha\partial^\alpha\Omega - \tfrac{1}{2}\Omega^{-4}\partial_\beta\partial^\beta\partial_\alpha\partial_\mu K_\nu{}^\alpha - \tfrac{1}{2}\Omega^{-4}\partial_\beta\partial^\beta\partial_\alpha\partial_\nu K_\mu{}^\alpha \\
&- 4\Omega^{-5}\partial_\alpha K_{\mu\nu}\partial_\beta\partial^\beta\partial^\alpha\Omega + \Omega^{-5}\partial^\alpha\Omega\partial_\beta\partial^\beta\partial_\mu K_{\nu\alpha} + \Omega^{-5}\partial^\alpha\Omega\partial_\beta\partial^\beta\partial_\nu K_{\mu\alpha} \\
&- \tfrac{4}{3}\Omega^{-5}\partial^\alpha\partial_\nu\Omega\partial_\beta\partial_\mu K_\alpha{}^\beta + \Omega^{-5}\partial_\alpha\partial^\alpha\Omega\partial_\beta\partial_\mu K_\nu{}^\beta - 3\Omega^{-6}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta\partial_\mu K_\nu{}^\beta \\
&- 6K_\nu{}^\beta\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\mu\partial_\alpha\Omega - 3K_{\nu\alpha}\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\mu\partial^\beta\Omega - \tfrac{4}{3}\Omega^{-5}\partial^\alpha\partial_\mu\Omega\partial_\beta\partial_\nu K_\alpha{}^\beta \\
&+ \Omega^{-5}\partial_\alpha\partial^\alpha\Omega\partial_\beta\partial_\nu K_\mu{}^\beta - 3\Omega^{-6}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta\partial_\nu K_\mu{}^\beta - 6K_\mu{}^\beta\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\nu\partial_\alpha\Omega \\
&- 3K_{\mu\alpha}\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\nu\partial^\beta\Omega - \tfrac{4}{3}\Omega^{-5}\partial^\alpha\Omega\partial_\beta\partial_\nu\partial_\mu K_\alpha{}^\beta - \tfrac{4}{3}\Omega^{-5}\partial_\alpha K^{\alpha\beta}\partial_\beta\partial_\nu\partial_\mu\Omega \\
&+ 4K_\alpha{}^\beta\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\nu\partial_\mu\Omega - 48\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta K_{\mu\nu}\partial^\beta\Omega + 60K_{\mu\nu}\Omega^{-8}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta\Omega\partial^\beta\Omega \\
&+ 12\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\alpha K_{\mu\nu}\partial^\beta\Omega - 48K_{\mu\nu}\Omega^{-7}\partial^\alpha\Omega\partial_\beta\partial_\alpha\Omega\partial^\beta\Omega - 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\mu K_{\nu\alpha}\partial^\beta\Omega \\
&- 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\nu K_{\mu\alpha}\partial^\beta\Omega + 24\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_{\mu\nu}\partial^\beta\partial_\alpha\Omega + K_{\nu\beta}\Omega^{-5}\partial^\beta\partial_\alpha\partial_\mu\partial^\alpha\Omega \\
&+ K_{\mu\beta}\Omega^{-5}\partial^\beta\partial_\alpha\partial_\nu\partial^\alpha\Omega + 2\Omega^{-5}\partial_\alpha\partial_\mu K_{\nu\beta}\partial^\beta\partial^\alpha\Omega + 2\Omega^{-5}\partial_\alpha\partial_\nu K_{\mu\beta}\partial^\beta\partial^\alpha\Omega \\
&- 4\Omega^{-5}\partial_\beta\partial_\alpha K_{\mu\nu}\partial^\beta\partial^\alpha\Omega + 6K_{\mu\nu}\Omega^{-6}\partial_\beta\partial_\alpha\Omega\partial^\beta\partial^\alpha\Omega - 6\Omega^{-6}\partial_\alpha K_{\nu\beta}\partial^\alpha\Omega\partial^\beta\partial_\mu\Omega \\
&+ 2\Omega^{-5}\partial_\alpha K_{\nu\beta}\partial^\beta\partial_\mu\partial^\alpha\Omega - 6\Omega^{-6}\partial_\alpha K_{\mu\beta}\partial^\alpha\Omega\partial^\beta\partial_\nu\Omega + 2\Omega^{-5}\partial_\alpha K_{\mu\beta}\partial^\beta\partial_\nu\partial^\alpha\Omega \\
&+ 2\eta_{\mu\nu}\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega\partial_\gamma K_\alpha{}^\gamma - 8\eta_{\mu\nu}\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial^\beta\Omega\partial_\gamma K_\beta{}^\gamma + 4\eta_{\mu\nu}\Omega^{-6}\partial^\alpha\Omega\partial^\beta\partial_\alpha\Omega\partial_\gamma K_\beta{}^\gamma \\
&- \tfrac{2}{3}\eta_{\mu\nu}\Omega^{-5}\partial^\beta\partial_\alpha\partial^\alpha\Omega\partial_\gamma K_\beta{}^\gamma + 2\eta_{\mu\nu}K_\beta{}^\gamma\Omega^{-6}\partial^\beta\partial^\alpha\Omega\partial_\gamma\partial_\alpha\Omega + 4\eta_{\mu\nu}\Omega^{-6}\partial^\alpha\Omega\partial^\beta\Omega\partial_\gamma\partial_\beta K_\alpha{}^\gamma \\
&- \tfrac{4}{3}\eta_{\mu\nu}\Omega^{-5}\partial^\beta\partial^\alpha\Omega\partial_\gamma\partial_\beta K_\alpha{}^\gamma - \tfrac{1}{3}\eta_{\mu\nu}\Omega^{-5}\partial_\alpha\partial^\alpha\Omega\partial_\gamma\partial_\beta K^{\beta\gamma} + \eta_{\mu\nu}\Omega^{-6}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\gamma\partial_\beta K^{\beta\gamma} \\
&+ \eta_{\mu\nu}K^{\beta\gamma}\Omega^{-6}\partial_\alpha\partial^\alpha\Omega\partial_\gamma\partial_\beta\Omega - 4\eta_{\mu\nu}K^{\beta\gamma}\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\gamma\partial_\beta\Omega - 16\eta_{\mu\nu}K_\alpha{}^\gamma\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\gamma\partial_\beta\Omega \\
&- \tfrac{2}{3}\eta_{\mu\nu}\Omega^{-5}\partial^\alpha\Omega\partial_\gamma\partial_\beta\partial_\alpha K^{\beta\gamma} + 2\eta_{\mu\nu}K^{\beta\gamma}\Omega^{-6}\partial^\alpha\Omega\partial_\gamma\partial_\beta\partial_\alpha\Omega + \eta_{\mu\nu}\Omega^{-6}\partial^\alpha\Omega\partial^\beta\Omega\partial_\gamma\partial^\gamma K_{\alpha\beta}
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{1}{3}\eta_{\mu\nu}\Omega^{-5}\partial^\beta\partial^\alpha\Omega\partial_\gamma\partial^\gamma K_{\alpha\beta} - 4\eta_{\mu\nu}K_{\alpha\beta}\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\gamma\partial^\gamma\Omega - \tfrac{2}{3}\eta_{\mu\nu}\Omega^{-5}\partial^\alpha\Omega\partial_\gamma\partial^\gamma\partial_\beta K_\alpha{}^\beta \\
&+ \tfrac{1}{6}\eta_{\mu\nu}\Omega^{-4}\partial_\gamma\partial^\gamma\partial_\beta\partial_\alpha K^{\alpha\beta} + 20\eta_{\mu\nu}K_{\beta\gamma}\Omega^{-8}\partial_\alpha\Omega\partial^\alpha\Omega\partial^\beta\Omega\partial^\gamma\Omega - 8\eta_{\mu\nu}\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\gamma K_{\alpha\beta}\partial^\gamma\Omega \\
&+ 2\eta_{\mu\nu}K_{\alpha\gamma}\Omega^{-6}\partial^\alpha\Omega\partial^\gamma\partial_\beta\partial^\beta\Omega + 2\eta_{\mu\nu}\Omega^{-6}\partial_\alpha K_{\beta\gamma}\partial^\alpha\Omega\partial^\gamma\partial^\beta\Omega + 4\eta_{\mu\nu}\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_{\alpha\gamma}\partial^\gamma\partial^\beta\Omega \\
&- \tfrac{1}{3}\eta_{\mu\nu}K_{\beta\gamma}\Omega^{-5}\partial^\gamma\partial^\beta\partial_\alpha\partial^\alpha\Omega - \tfrac{2}{3}\eta_{\mu\nu}\Omega^{-5}\partial_\alpha K_{\beta\gamma}\partial^\gamma\partial^\beta\partial^\alpha\Omega + 4\Omega^{-6}\partial^\alpha\Omega\partial^\beta\partial_\nu\Omega\partial_\mu K_{\alpha\beta} \\
&- \tfrac{2}{3}\Omega^{-5}\partial_\beta\partial_\nu\partial_\alpha\Omega\partial_\mu K^{\alpha\beta} - 3\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega\partial_\mu K_{\nu\alpha} + 12\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial^\beta\Omega\partial_\mu K_{\nu\beta} \\
&- 6\Omega^{-6}\partial^\alpha\Omega\partial^\beta\partial_\alpha\Omega\partial_\mu K_{\nu\beta} + \Omega^{-5}\partial^\beta\partial_\alpha\partial^\alpha\Omega\partial_\mu K_{\nu\beta} + 4\Omega^{-6}\partial^\alpha\partial_\nu\Omega\partial_\beta K_\alpha{}^\beta\partial_\mu\Omega \\
&- 3\Omega^{-6}\partial_\alpha\partial^\alpha\Omega\partial_\beta K_\nu{}^\beta\partial_\mu\Omega + 12\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta K_\nu{}^\beta\partial_\mu\Omega - 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\alpha K_\nu{}^\beta\partial_\mu\Omega \\
&+ 24K_\nu{}^\beta\Omega^{-7}\partial^\alpha\Omega\partial_\beta\partial_\alpha\Omega\partial_\mu\Omega - \tfrac{2}{3}\Omega^{-5}\partial_\beta\partial_\alpha\partial_\nu K^{\alpha\beta}\partial_\mu\Omega - 3\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial^\beta K_{\nu\alpha}\partial_\mu\Omega \\
&+ 12K_{\nu\alpha}\Omega^{-7}\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega\partial_\mu\Omega + \Omega^{-5}\partial_\beta\partial^\beta\partial_\alpha K_\nu{}^\alpha\partial_\mu\Omega + 4\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\nu K_\alpha{}^\beta\partial_\mu\Omega \\
&+ 2K^{\alpha\beta}\Omega^{-6}\partial_\beta\partial_\nu\partial_\alpha\Omega\partial_\mu\Omega - 60K_{\nu\beta}\Omega^{-8}\partial_\alpha\Omega\partial^\alpha\Omega\partial^\beta\Omega\partial_\mu\Omega + 24\Omega^{-7}\partial^\alpha\Omega\partial_\beta K_{\nu\alpha}\partial^\beta\Omega\partial_\mu\Omega \\
&- 3K_{\nu\beta}\Omega^{-6}\partial^\beta\partial_\alpha\partial^\alpha\Omega\partial_\mu\Omega - 6\Omega^{-6}\partial_\alpha K_{\nu\beta}\partial^\beta\partial^\alpha\Omega\partial_\mu\Omega - 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_\nu{}^\beta\partial_\mu\partial_\alpha\Omega \\
&- 6K_{\nu\beta}\Omega^{-6}\partial^\beta\partial^\alpha\Omega\partial_\mu\partial_\alpha\Omega - 3K_\nu{}^\beta\Omega^{-6}\partial_\alpha\partial^\alpha\Omega\partial_\mu\partial_\beta\Omega + 12K_\nu{}^\beta\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\mu\partial_\beta\Omega \\
&+ 24K_{\nu\alpha}\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\mu\partial_\beta\Omega - 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_{\nu\alpha}\partial_\mu\partial^\beta\Omega + 4\Omega^{-6}\partial^\alpha\Omega\partial^\beta\partial_\mu\Omega\partial_\nu K_{\alpha\beta} \\
&- 8\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\mu\Omega\partial_\nu K_{\alpha\beta} + 2\Omega^{-6}\partial^\beta\partial^\alpha\Omega\partial_\mu\Omega\partial_\nu K_{\alpha\beta} - \tfrac{2}{3}\Omega^{-5}\partial_\beta\partial_\mu\partial_\alpha\Omega\partial_\nu K^{\alpha\beta} \\
&- 3\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega\partial_\nu K_{\mu\alpha} + 12\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial^\beta\Omega\partial_\nu K_{\mu\beta} - 6\Omega^{-6}\partial^\alpha\Omega\partial^\beta\partial_\alpha\Omega\partial_\nu K_{\mu\beta} \\
&+ \Omega^{-5}\partial^\beta\partial_\alpha\partial^\alpha\Omega\partial_\nu K_{\mu\beta} + 4\Omega^{-6}\partial^\alpha\partial_\mu\Omega\partial_\beta K_\alpha{}^\beta\partial_\nu\Omega - 3\Omega^{-6}\partial_\alpha\partial^\alpha\Omega\partial_\beta K_\mu{}^\beta\partial_\nu\Omega \\
&+ 12\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\beta K_\mu{}^\beta\partial_\nu\Omega - 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\alpha K_\mu{}^\beta\partial_\nu\Omega + 24K_\mu{}^\beta\Omega^{-7}\partial^\alpha\Omega\partial_\beta\partial_\alpha\Omega\partial_\nu\Omega \\
&- \tfrac{2}{3}\Omega^{-5}\partial_\beta\partial_\alpha\partial_\mu K^{\alpha\beta}\partial_\nu\Omega - 3\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial^\beta K_{\mu\alpha}\partial_\nu\Omega + 12K_{\mu\alpha}\Omega^{-7}\partial^\alpha\Omega\partial_\beta\partial^\beta\Omega\partial_\nu\Omega \\
&+ \Omega^{-5}\partial_\beta\partial^\beta\partial_\alpha K_\mu{}^\alpha\partial_\nu\Omega + 4\Omega^{-6}\partial^\alpha\Omega\partial_\beta\partial_\mu K_\alpha{}^\beta\partial_\nu\Omega + 2K^{\alpha\beta}\Omega^{-6}\partial_\beta\partial_\mu\partial_\alpha\Omega\partial_\nu\Omega \\
&- 60K_{\mu\beta}\Omega^{-8}\partial_\alpha\Omega\partial^\alpha\Omega\partial^\beta\Omega\partial_\nu\Omega + 24\Omega^{-7}\partial^\alpha\Omega\partial_\beta K_{\mu\alpha}\partial^\beta\Omega\partial_\nu\Omega - 3K_{\mu\beta}\Omega^{-6}\partial^\beta\partial_\alpha\partial^\alpha\Omega\partial_\nu\Omega \\
&- 6\Omega^{-6}\partial_\alpha K_{\mu\beta}\partial^\beta\partial^\alpha\Omega\partial_\nu\Omega - 8\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\mu K_{\alpha\beta}\partial_\nu\Omega + 2\Omega^{-6}\partial^\beta\partial^\alpha\Omega\partial_\mu K_{\alpha\beta}\partial_\nu\Omega \\
&- 16\Omega^{-7}\partial^\alpha\Omega\partial_\beta K_\alpha{}^\beta\partial_\mu\Omega\partial_\nu\Omega + 2\Omega^{-6}\partial_\beta\partial_\alpha K^{\alpha\beta}\partial_\mu\Omega\partial_\nu\Omega - 8K^{\alpha\beta}\Omega^{-7}\partial_\beta\partial_\alpha\Omega\partial_\mu\Omega\partial_\nu\Omega \\
&+ 40K_{\alpha\beta}\Omega^{-8}\partial^\alpha\Omega\partial^\beta\Omega\partial_\mu\Omega\partial_\nu\Omega - 16K_\alpha{}^\beta\Omega^{-7}\partial^\alpha\Omega\partial_\mu\partial_\beta\Omega\partial_\nu\Omega - 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_\mu{}^\beta\partial_\nu\partial_\alpha\Omega \\
&- 6K_{\mu\beta}\Omega^{-6}\partial^\beta\partial^\alpha\Omega\partial_\nu\partial_\alpha\Omega - 3K_\mu{}^\beta\Omega^{-6}\partial_\alpha\partial^\alpha\Omega\partial_\nu\partial_\beta\Omega + 12K_\mu{}^\beta\Omega^{-7}\partial_\alpha\Omega\partial^\alpha\Omega\partial_\nu\partial_\beta\Omega \\
&+ 24K_{\mu\alpha}\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\nu\partial_\beta\Omega - 16K_\alpha{}^\beta\Omega^{-7}\partial^\alpha\Omega\partial_\mu\Omega\partial_\nu\partial_\beta\Omega + 4K^{\alpha\beta}\Omega^{-6}\partial_\mu\partial_\alpha\Omega\partial_\nu\partial_\beta\Omega \\
&- 6\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_{\mu\alpha}\partial_\nu\partial^\beta\Omega + 2\Omega^{-6}\partial^\alpha\Omega\partial^\beta\Omega\partial_\nu\partial_\mu K_{\alpha\beta} - \tfrac{2}{3}\Omega^{-5}\partial^\beta\partial^\alpha\Omega\partial_\nu\partial_\mu K_{\alpha\beta} \\
&+ 4\Omega^{-6}\partial^\alpha\Omega\partial_\beta K_\alpha{}^\beta\partial_\nu\partial_\mu\Omega - \tfrac{2}{3}\Omega^{-5}\partial_\beta\partial_\alpha K^{\alpha\beta}\partial_\nu\partial_\mu\Omega + 2K^{\alpha\beta}\Omega^{-6}\partial_\beta\partial_\alpha\Omega\partial_\nu\partial_\mu\Omega \\
&- 8K_{\alpha\beta}\Omega^{-7}\partial^\alpha\Omega\partial^\beta\Omega\partial_\nu\partial_\mu\Omega,
\end{aligned}
\tag{2}
$$

where $K_{\mu\nu} = h_{\mu\nu} - (1/4)g_{\mu\nu}g^{\alpha\beta}h_{\alpha\beta}$ is the traceless part of the fluctuation. Despite its 151 terms we can rewrite this expression identically as the compact

$$\begin{aligned}\delta W_{\mu\nu} &= \frac{1}{2}\Omega^{-2}\bigg(\partial_\sigma\partial^\sigma\partial_\tau\partial^\tau[\Omega^{-2}K_{\mu\nu}] - \partial_\sigma\partial^\sigma\partial_\mu\partial^\alpha[\Omega^{-2}K_{\alpha\nu}] - \partial_\sigma\partial^\sigma\partial_\nu\partial^\alpha[\Omega^{-2}K_{\alpha\mu}] \\ &+ \frac{2}{3}\partial_\mu\partial_\nu\partial^\alpha\partial^\beta[\Omega^{-2}K_{\alpha\beta}] + \frac{1}{3}\eta_{\mu\nu}\partial_\sigma\partial^\sigma\partial^\alpha\partial^\beta[\Omega^{-2}K_{\alpha\beta}]\bigg). \end{aligned} \tag{3}$$

Then, on introducing the transverse-traceless projector we can write $\delta W_{\mu\nu}$ even more compactly as

$$\delta W_{\mu\nu} = \frac{1}{2}\Omega^{-2}\eta^{\sigma\rho}\eta^{\alpha\beta}\partial_\sigma\partial_\rho\partial_\alpha\partial_\beta[\Omega^{-2}h_{\mu\nu}]^{T\theta}, \tag{4}$$

Quantum Conformal Gravity

While I have described properties of conformal gravity at the classical level, this would all be irrelevant if the theory did not also make sense at the quantum level. Unlike Einstein gravity the conformal gravity theory is renormalizable, with its radiative corrections being under control. (The second-order Einstein theory propagator behaves as $1/k^2$ while the fourth-order conformal gravity propagator behaves as the much more convergent $1/k^4$.) However, conformal gravity had been thought to not be a probability conserving unitary theory as its Hilbert space was thought to possess states of negative Dirac norm (the overlap of a ket state with its Hermitian conjugate bra state). To see the issue consider the propagator

$$\frac{1}{k^4} = \lim_{M^2 \to 0} \frac{1}{M^2}\left(\frac{1}{k^2 - M^2} - \frac{1}{k^2}\right) \tag{5}$$

Some of the poles have residues that are negative, to thus seemingly be associated with ghost states with negative norm.

However, the propagator is a c-number, a matrix element of quantum field theory operators, and one cannot actually make any claims about the nature of the states in the Hilbert space just by looking at a c-number. Rather one has to explicitly construct the quantum Hilbert space.

Now as written, because of the $1/M^2$ factor the $M^2 \to 0$ limit might actually be singular, and in a collaboration with Aharon Davidson in 2000 we constructed the quantum Hilbert space and showed that the limit was singular. Thus anything inferred about the $M^2 \neq 0$ theory does not necessarily carry over to the $M^2 = 0$ one. And Aharon and I were able to show that even if the $M^2 \neq 0$ theory has states of negative norm states (something we thought to be the case), the $M^2 = 0$ theory did not. Thus conformal gravity did not have a ghost problem.

However, what happens before we take the limit. In a collaboration with Carl Bender in 2008 we showed that even before taking the $M^2 \to 0$ limit the structure of the Hilbert space was such that even then there were no negative norm states. In particular we showed that the quantum Hamiltonian was not Hermitian, rather it was in the class of non-Hermitian but PT symmetric theories that had been developed by Carl (P is parity, T is time reversal, an antilinear operator). In consequence, we have to continue the theory into the complex plane, and in so doing we found that the appropriate norm was the overlap of a ket state with its PT conjugate bra state, and this norm is positive. Thus the theory is unitary.

Now giving up Hermiticity might appear to be unacceptable. However, while Hermiticity implies reality of eigenvalues, there is no converse theorem that says some of the eigenvalues of a non-Hermitian Hamiltonian must be complex. Hermiticity is only **SUFFICIENT** to secure reality but not necessary. I showed that the necessary requirement is that the Hamiltonian have an antilinear symmetry such as PT.

Conformal gravity is thus a consistent quantum theory of gravity, and if it turns out to be the correct theory of quantum gravity, then one of the four fundamental forces in nature would be a PT theory.

But what about the fact that the Dirac spinor is reducible under the Lorentz group

Underpinning conformal symmetry is the conformal group, a complex not a real group. The Lorentz group is a subgroup of it and the other generators of the conformal group then link the two-component left-handed Weyl spinors and the two-component right-handed Weyl spinors; with the four-component Dirac spinor now being irreducible. Thus the conformal group must be the fundamental group of nature, and not the Lorentz group. And if it is, then right-handed neutrinos must exist, and gravity must have a conformal structure.

In addition, the conformal group contains the PT symmetry generator, and this generator extends the real Lorentz group to the complex one, with the complex Lorentz group (so important for PT symmetry) thus lying within the conformal group.

Thus the questions I asked in graduate school have all come together in the research I have done over the years, research I could not have done without the continuing and never waning support I have constantly received from my colleagues and collaborators. And all the more so given the fact that my work is so non-mainstream (I give up both Einstein gravity and Hermiticity.)

Finally, I must thank my wife Fay and my children Michael and Alexa for having put up with such an iconoclast for so long.

SO TO ALL OF YOU I SAY: THANK YOU, THANK YOU, THANK YOU.